

Many Logics, One Methodology

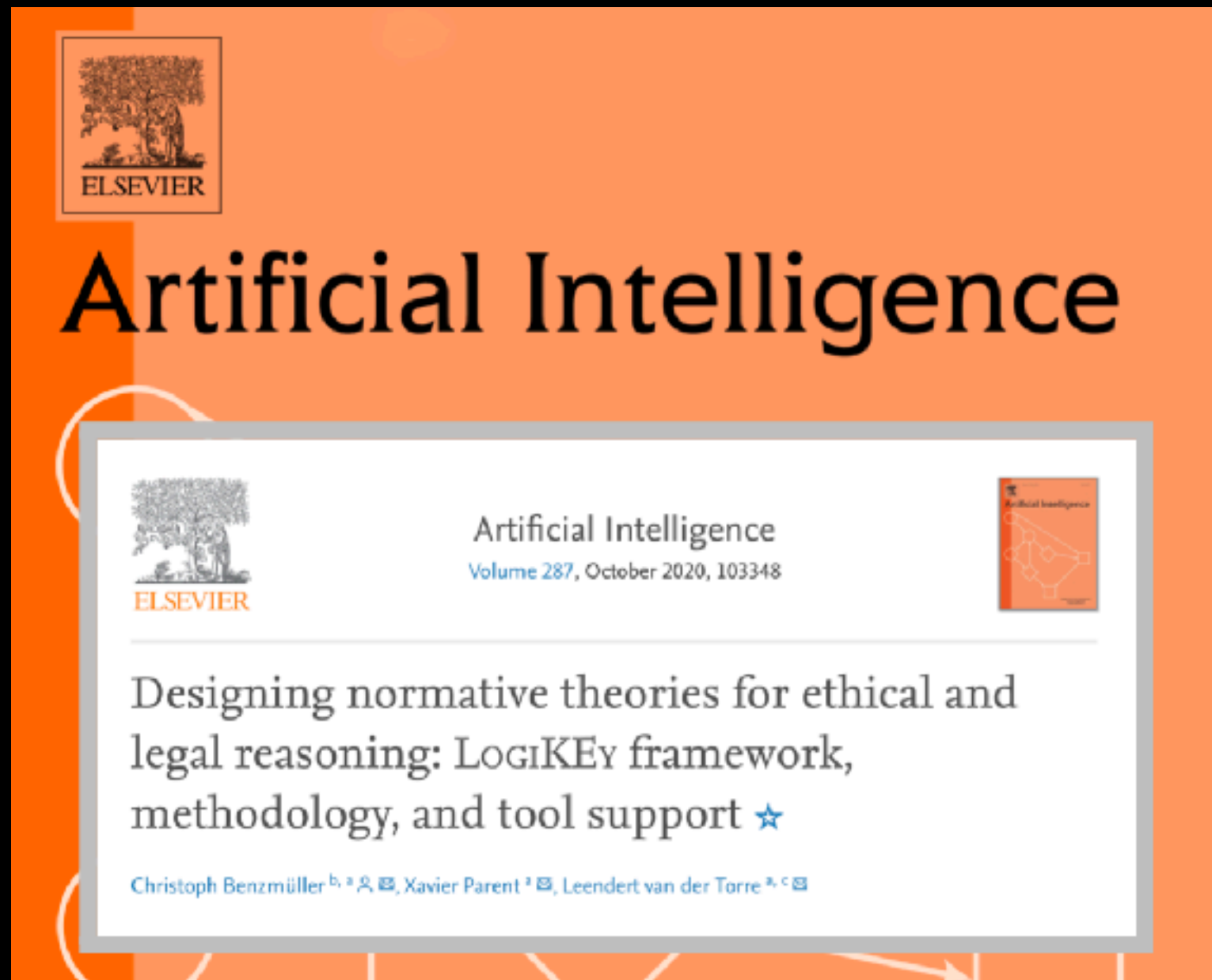
— LogiKEy —

Christoph Benz Müller
& Luca Pasetto

ESSLI 2026

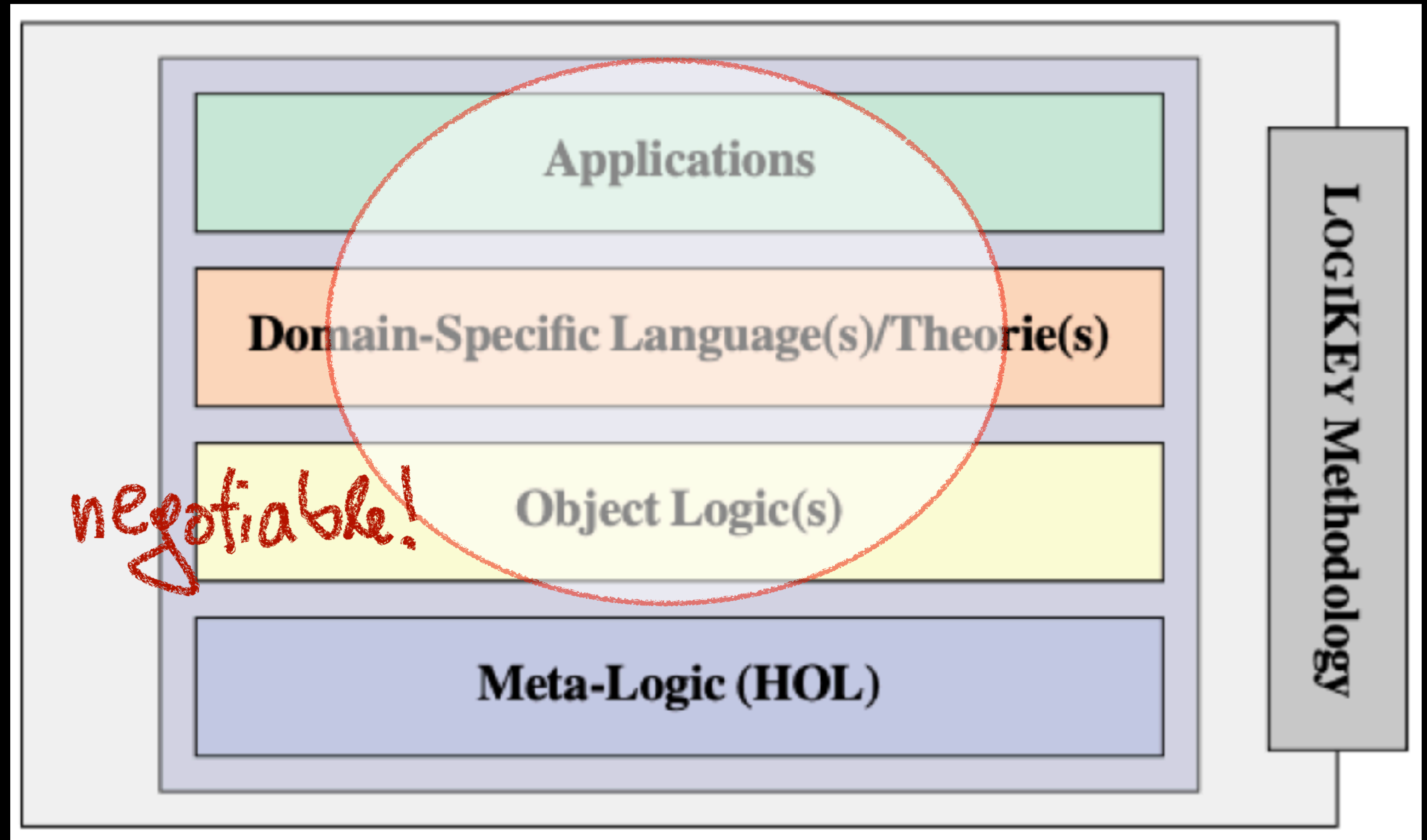


LogiKEy: Logic-pluralistic KR&R Methodology



2020

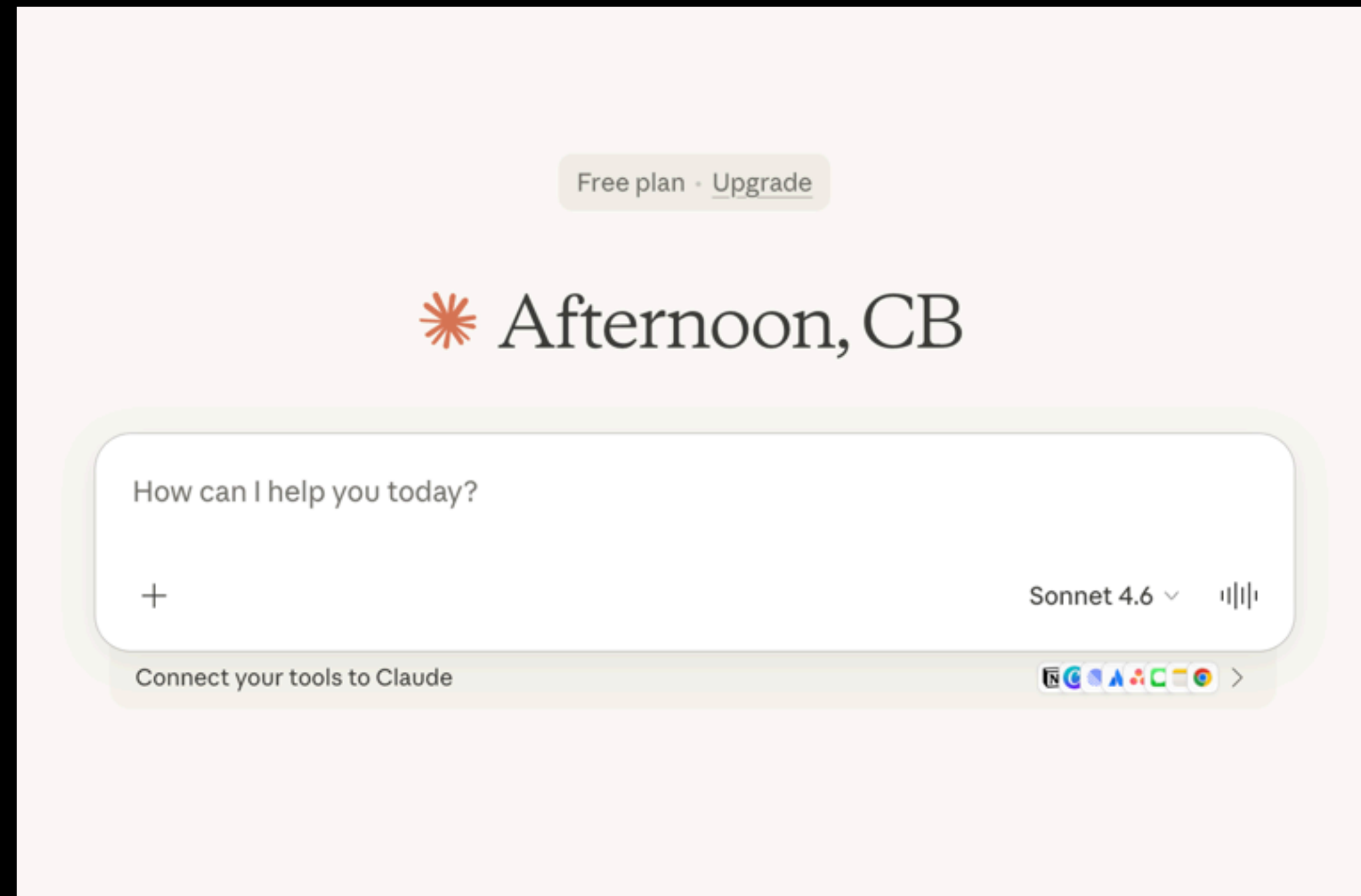
The term “**LogiKEy**” was introduced in this article with a focus on normative reasoning.



LogiKEy prefers **shallow** embeddings, but equally well supports **deep embeddings**.

Informal and Formal Reasoning

New world of LLMs



Even toddlers can use this

The world of KR&R in symbolic AI



Lost users in a zoo of logics and tools
(LogiKEy: one tool for all logics ... ?)

Logical/Mathematical Imperialism

Choose some logical foundation and don't question them ever again.

For example, commit to a priori assumptions like:

$\exists x . x = x$	existence of objects, non-empty types
$\exists x . x = 1/0$	terms always denote, undefinedness exists
$\forall \varphi . \varphi \vee \neg \varphi$	law of excluded middle, classical logic
$\forall \varphi \psi . (\varphi \equiv \psi) \supset (\varphi = \psi)$	boolean extensionality
$\forall \varphi \psi . (\forall x . \varphi x = \psi x) \supset (\varphi = \psi)$	functional extensionality
If $H \vdash \varphi$ then $H, \psi \vdash \varphi$	monotonicity
...	

Such choices are typically made in maths formalisation projects.

They might be completely unnacceptable for formalisations in philosophy, law, ...

Logical/Mathematical Pluralism — LogiKEy

Make logical foundations flexible and negotiable.

Support reflection on them within proof assistants.

$\exists x . x = x$

$\exists x . x = 1/0$

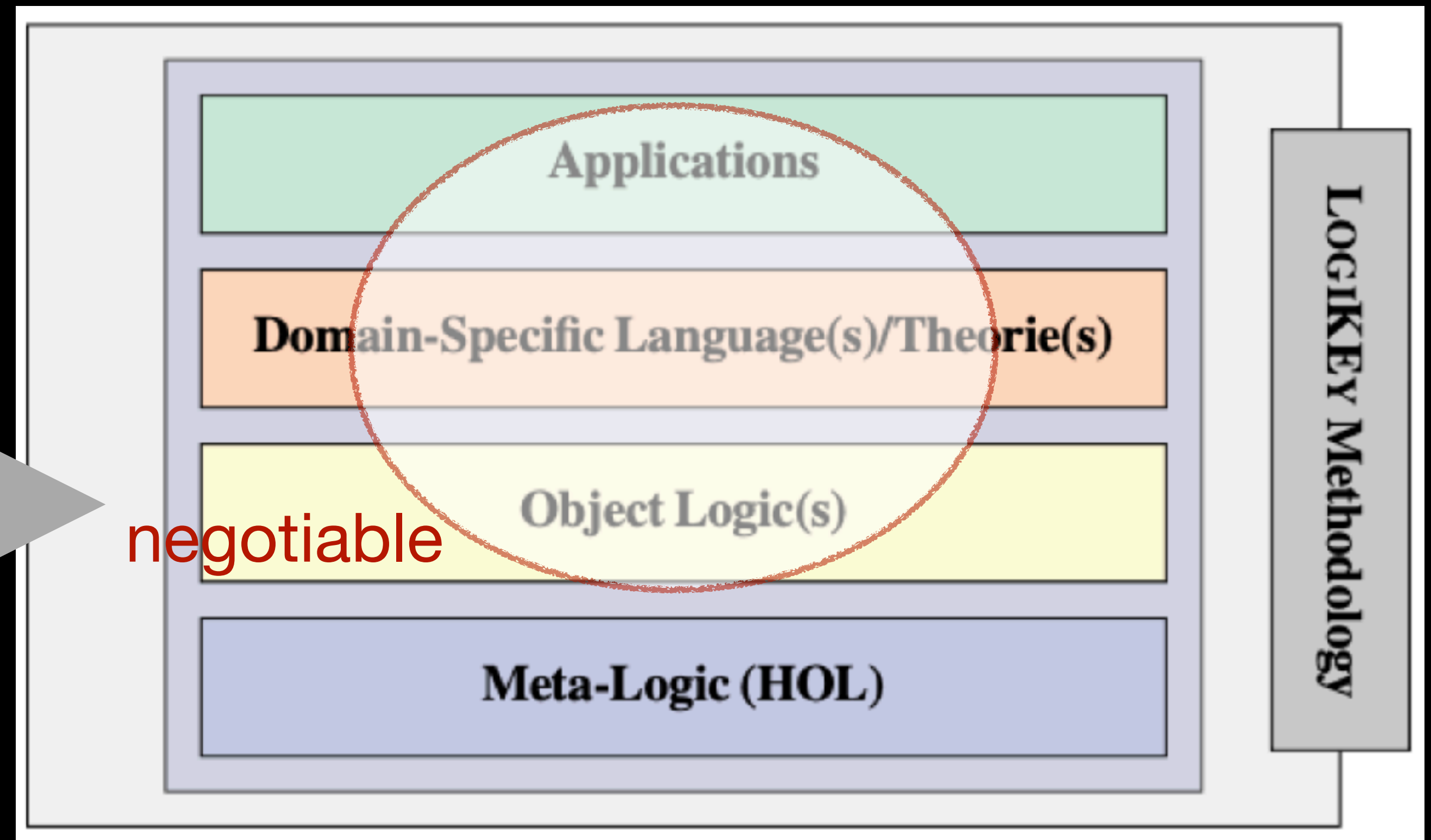
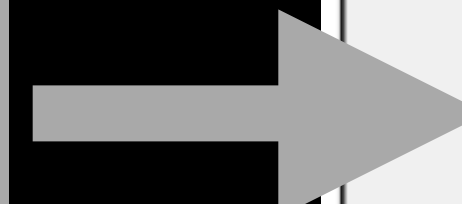
$\forall \varphi . \varphi \vee \neg \varphi$

$\forall \varphi \psi . (\varphi \equiv \psi) \supset (\varphi = \psi)$

$\forall \varphi \psi . (\forall x . \varphi x = \psi x) \supset (\varphi = \psi)$

If $H \vdash \varphi$ then $H, \psi \vdash \varphi$

...



This way proof assistants may support formalisations also in philosophy, law, ...

Logical/Mathematical Pluralism — LogiKEy

Make logical foundations flexible and negotiable.

Support reflection on them within proof assistants.

~~$\exists x. x = x$~~

~~$\exists x. x = 1/0$~~

Free Logic

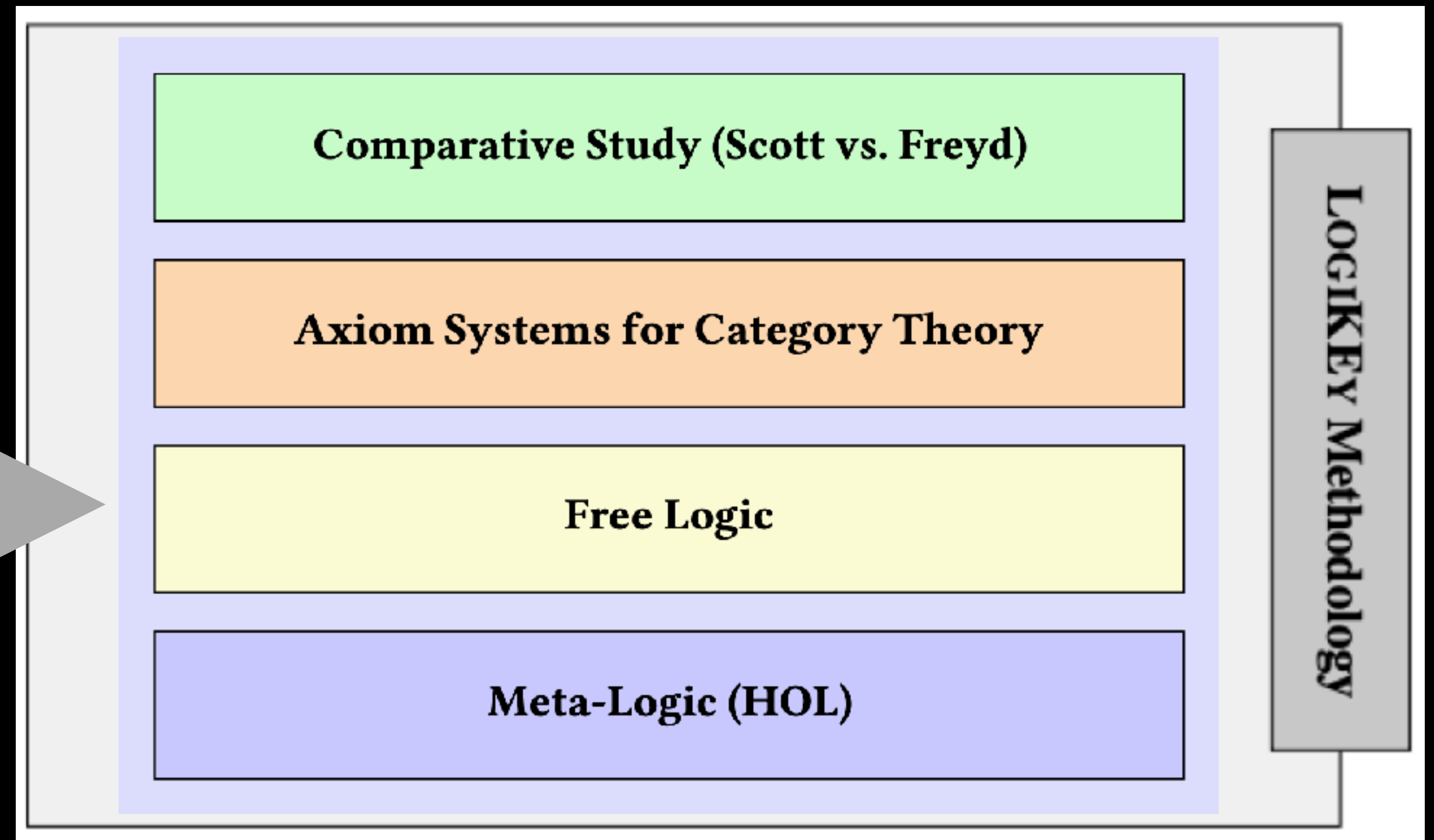
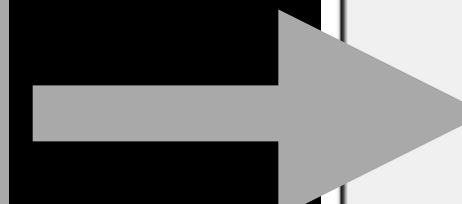
$\forall \varphi. \varphi \vee \neg \varphi$

$\forall \varphi \psi. (\varphi \equiv \psi) \supset (\varphi = \psi)$

$\forall \varphi \psi. (\forall x. \varphi x = \psi x) \supset (\varphi = \psi)$

If $H \vdash \varphi$ then $H, \psi \vdash \varphi$

...



Logical/Mathematical Pluralism — LogiKEy

Make logical foundations flexible and negotiable.

Support reflection on them within proof assistants.

$\exists x. x = x$

$\exists x. x = 1/0$

Free Logic

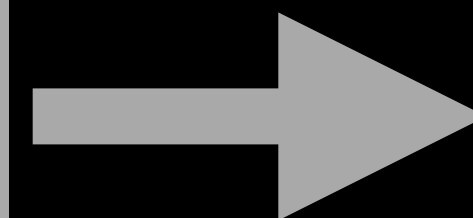
$\forall \varphi. \varphi \vee \neg \varphi$

$\forall \varphi \psi. (\varphi \equiv \psi) \supset (\varphi = \psi)$

$\forall \varphi \psi. (\forall x. \varphi x = \psi x) \supset (\varphi = \psi)$

If $H \vdash \varphi$ then $H, \psi \vdash \varphi$

...



Home > Journal of Automated Reasoning > Article

Automating Free Logic in HOL, with an Experimental Application in Category Theory

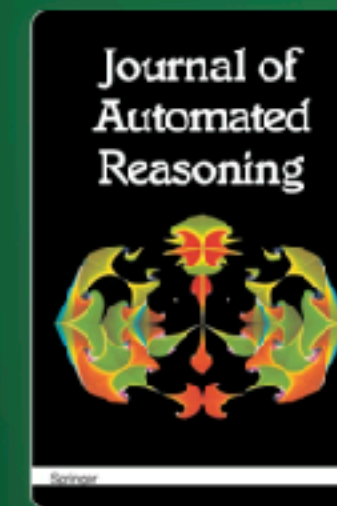
Published: 01 January 2019
Volume 64, pages 53–72, (2020) [Cite this article](#)

Access provided by Otto-Friedrich-Universität Bamberg

[Download PDF](#)

[Save article](#) [View saved research](#)

[Christoph Benzmüller](#) & [Dana S. Scott](#)



Journal of Automated Reasoning

[Aims and scope](#)

Home > Intelligent Computer Mathematics > Conference paper

Category Theory in Isabelle/HOL as a Basis for Meta-logical Investigation

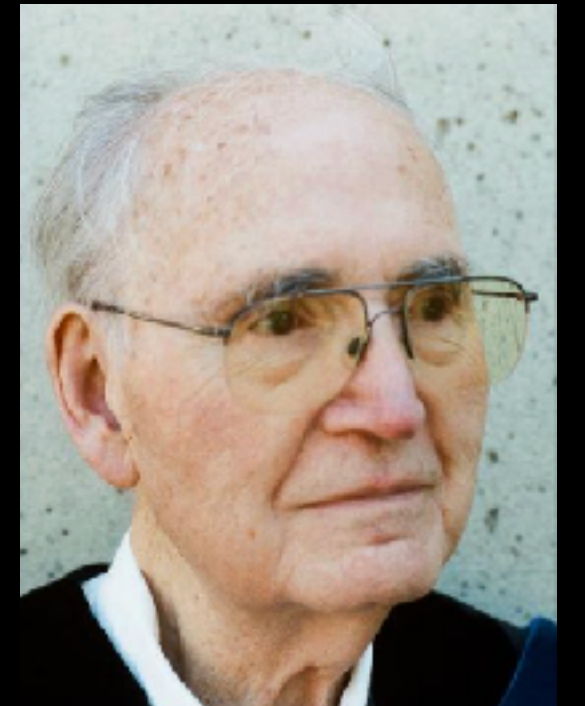
Conference paper | First Online: 28 August 2023
pp 69–83 | [Cite this conference paper](#)

Access provided by Otto-Friedrich-Universität Bamberg

[Book PDF](#)

[Book EPUB](#)

[Jonas Bayer](#), [Alexey Gonus](#), [Christoph Benzmüller](#) & [Dana S. Scott](#)

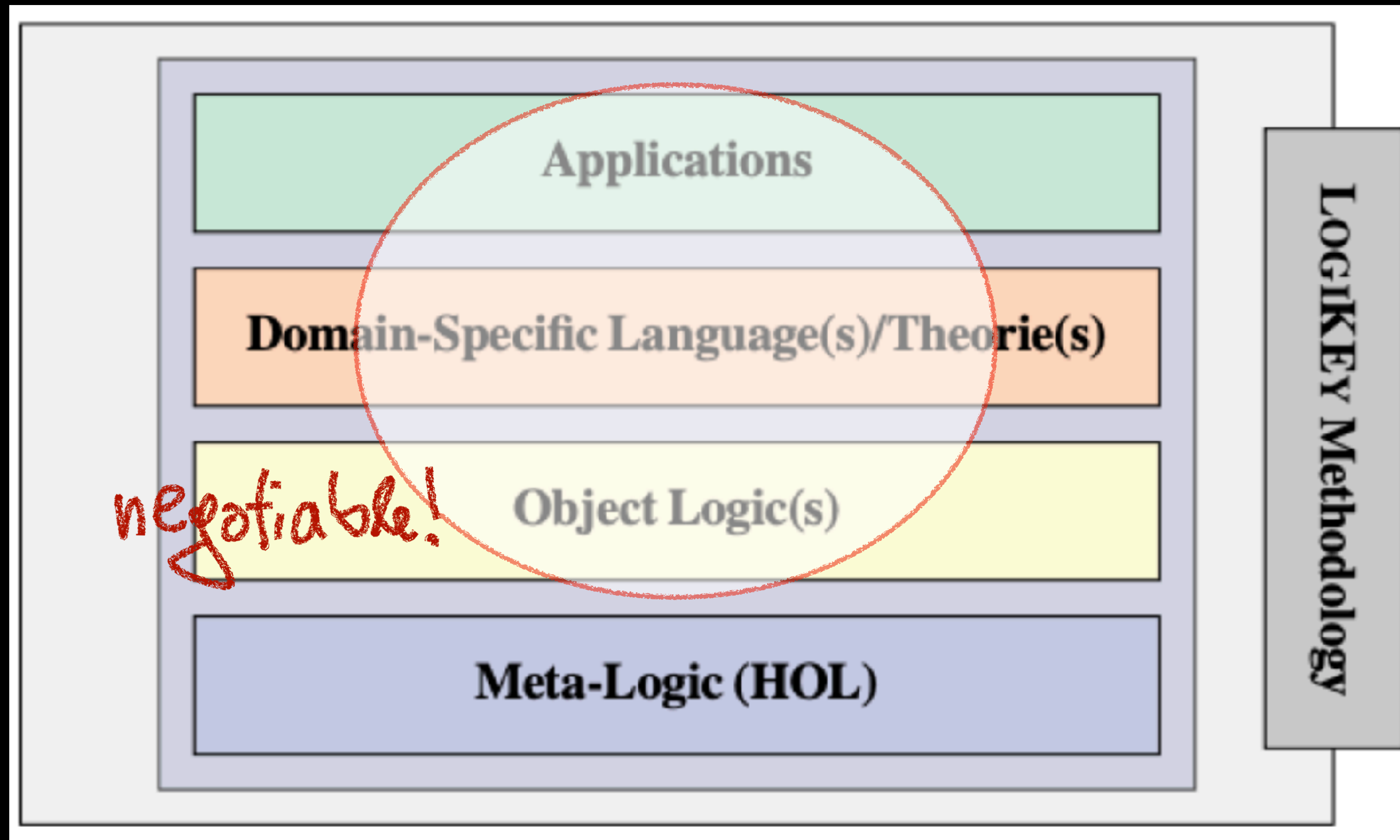


jww: Dana Scott



Intelligent
Computer
Mathematics
(CICM 2023)

LogiKEy: Logic-pluralistic KR&R Methodology

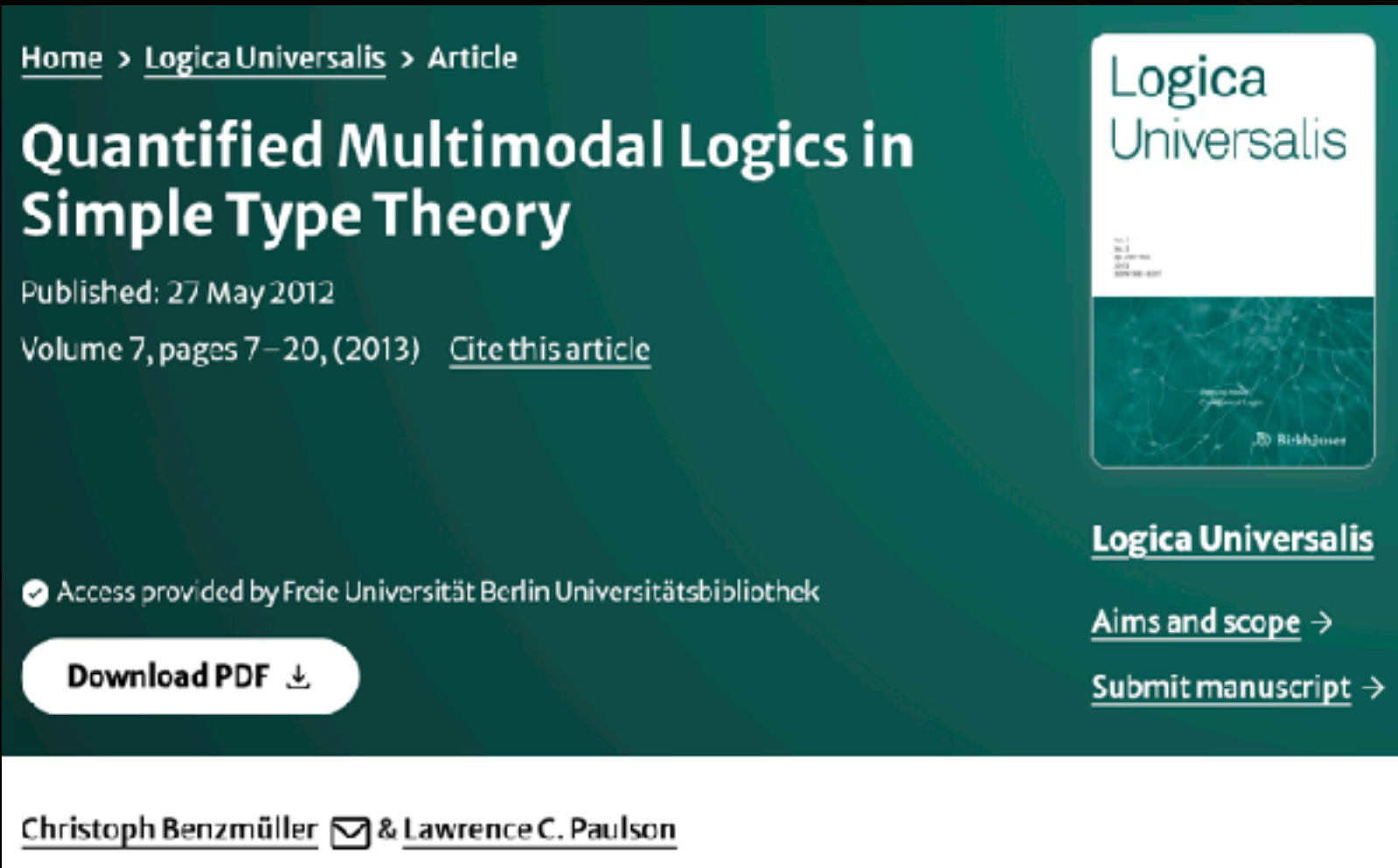
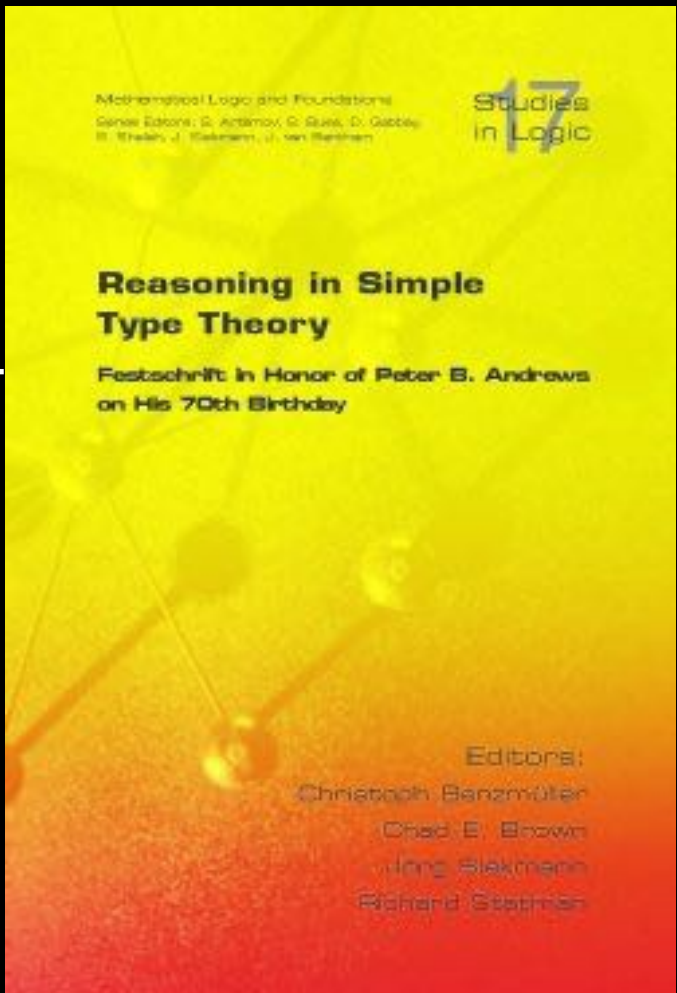


Origins: 2007-2008

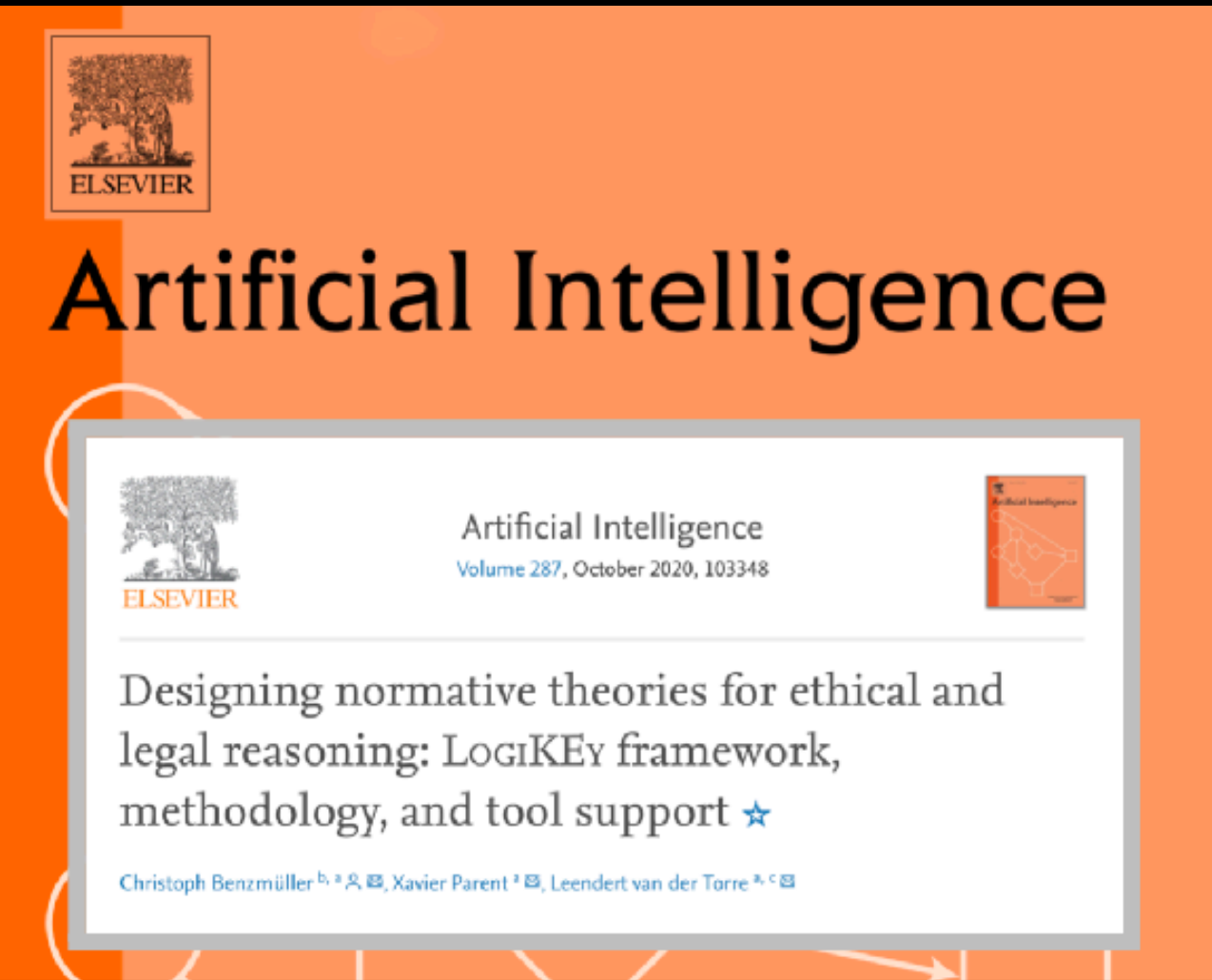
LogiKEy: Logic-pluralistic KR&R Methodology

Selection of logic embeddings:

- Multimodal Logics [Andrews Festschrift, 2008] jww
Larry Paulson
- Access Control Logic [SEP, 2009]
- Intuitionistic Logic [LJ IGPL, 2010]
- Quantified Multimodal Logic [Log Univers, 2012]
- Quantified Conditional Logic [J Phil Log, 2016]
- Multivalued Logic [LLP, 2016]
- Free Logic [JAR, 2020]
- Dyadic Deontic Logic [AIJ, 2020] L. van der Torre & X. Parent
jww
- (Hyper-)Intensional HO Modal Logic [RSL, 2020]
- Public Announcement Logic [JLC, 2022]
- Logic of Right to Know [ECAI, 2025]
- ...



jww: Larry Paulson



Term
LogiKEy
introduced
in 2020

LogiKEy: Logic-pluralistic KR&R Methodology

Selection of logic embeddings:

- Multimodal Logics [Andrews Festschrift, 2008]
- Access Control Logic [SEP, 2009]
- Intuitionistic Logic [LJ IGPL, 2010]
- Quantified Multimodal Logic [Log Univers, 2012]
- Quantified Conditional Logic [J Phil Log, 2016]
- Multivalued Logic [LLP, 2016]
- Free Logic [JAR, 2020]
- Dyadic Deontic Logic [AIJ, 2020] [JANCL, 2024]
- (Hyper-)Intensional HO Modal Logic [RSL, 2020]
- Public Announcement Logic [JLC, 2022]
- Logic of Right to Know [ECAI, 2025]
- ...

[Home](#) > [Monatshefte für Mathematik](#) > [Article](#)

Notes on Gödel's and Scott's variants of the ontological argument

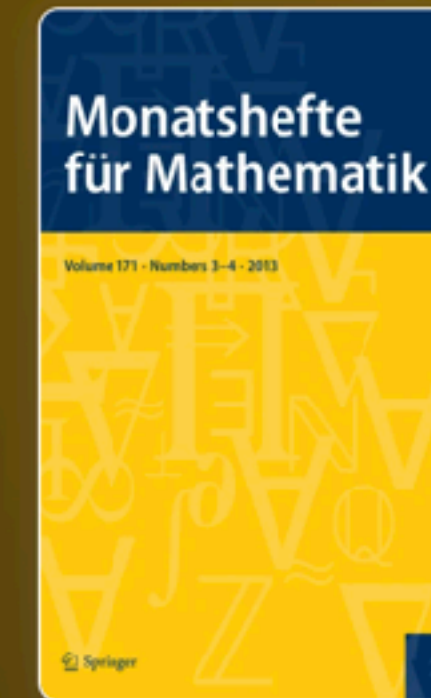
[Open access](#) | Published: 21 April 2025

Volume 208, pages 569–611, (2025) [Cite this article](#)

✓ You have full access to this [open access](#) article

[Download PDF](#) ↓

[Save article](#) [View saved research](#) >



[Monatshefte für Mathematik](#)

[Aims and scope](#) →

[Submit manuscript](#) →



jww: Dana Scott

[Christoph Benz Müller](#) ✉ & [Dana Scott](#)

2025

6378 Accesses 2 Citations 4 Altmetric [Explore all metrics](#) →

Abstract

Notes on Kurt Gödel's modal ontological argument and Dana Scott's variant of it are presented. These remarks, supported by experimental studies with a proof assistant system for classical higher-order logic, implicitly answer some questions the authors have received over the last decade(s). In addition, some new insights resulting from the conducted experiments are reported.

LogiKEy@work in Computational Metaphysics

LogiKey@work in Computational Metaphysics

SPIEGEL ONLINE

WISSENSCHAFT

Login | Registrierung

Formel von Kurt Gödel: Mathematiker bestätigen Gottesbeweis

Von Tobias Hürter



Kurt Gödel (um das Jahr 1935): Der Mathematiker hielt seinen Gottesbeweis jahrzehntelang geheim



Formel von Kurt Gödel: Mathematiker bestätigen Gottesbeweis

Von Tobias Hürter

Holy Logic: Computer Scientists 'Prove' God Exists

By David Knight

Home | Video | Themen | Forum | English | DER SPIEGEL | SPIEGEL TV | Abo | Shop

SPIEGEL ONLINE

INTERNATIONAL

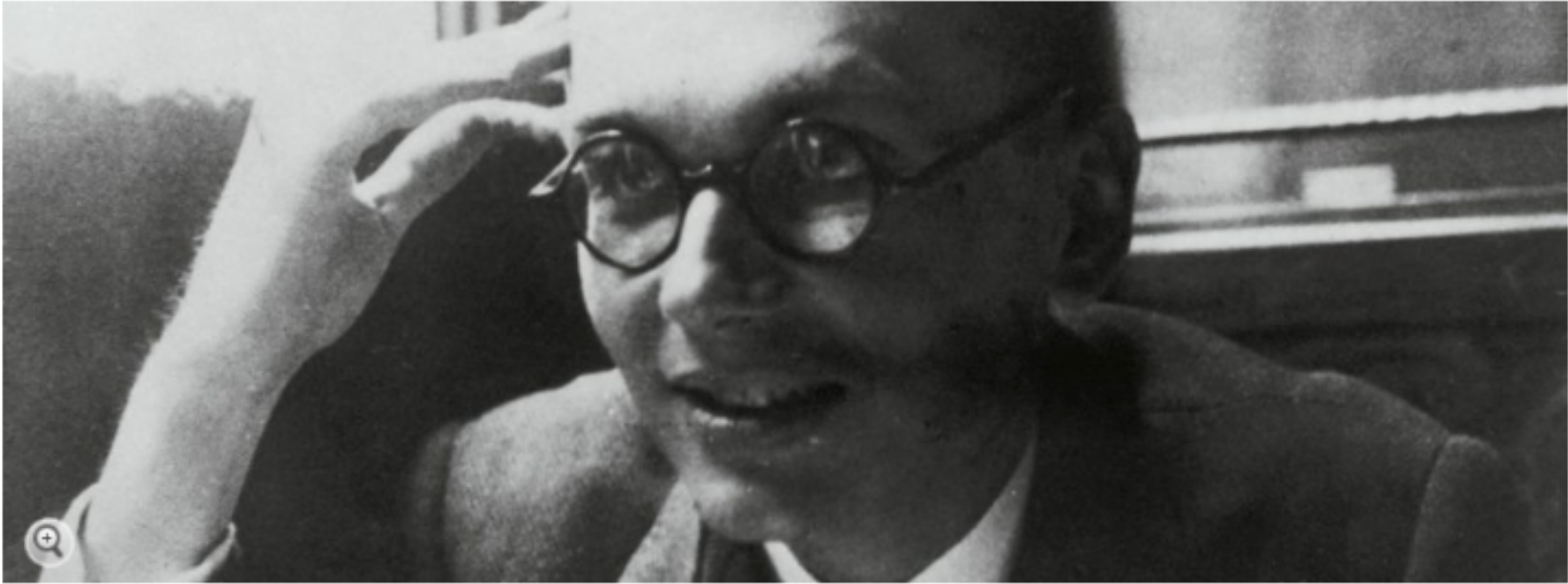
Sign in | Register

Front Page World Europe Germany Business Zeitgeist Newsletter

English Site > Germany > Science > Scientists Use Computer to Mathematically Prove Gödel God Theorem

Holy Logic: Computer Scientists 'Prove' God Exists

By David Knight



Austrian mathematician Kurt Gödel kept his proof of God's existence a secret for decades. Now two scientists say they have proven it mathematically using a computer.

Two scientists have formalized a theorem regarding the existence of God penned by mathematician Kurt Gödel. But the God angle is somewhat of a red herring -- the real step forward is the example it sets of how computers can make scientific progress simpler.



Bruno Woltzenlogel-Paleo

2013

MEDIA & CULTURE

Is God Real? Scientists 'Prove' His Existence With Godel's Theory And MacBooks

HOME / SCIENCE NEWS

Researchers say they used MacBook to prove Goedel's God theorem

God exists, say Apple fanboy scientists

With the help of just one MacBook, two Germans formalize a theorem that confirms the existence of God.

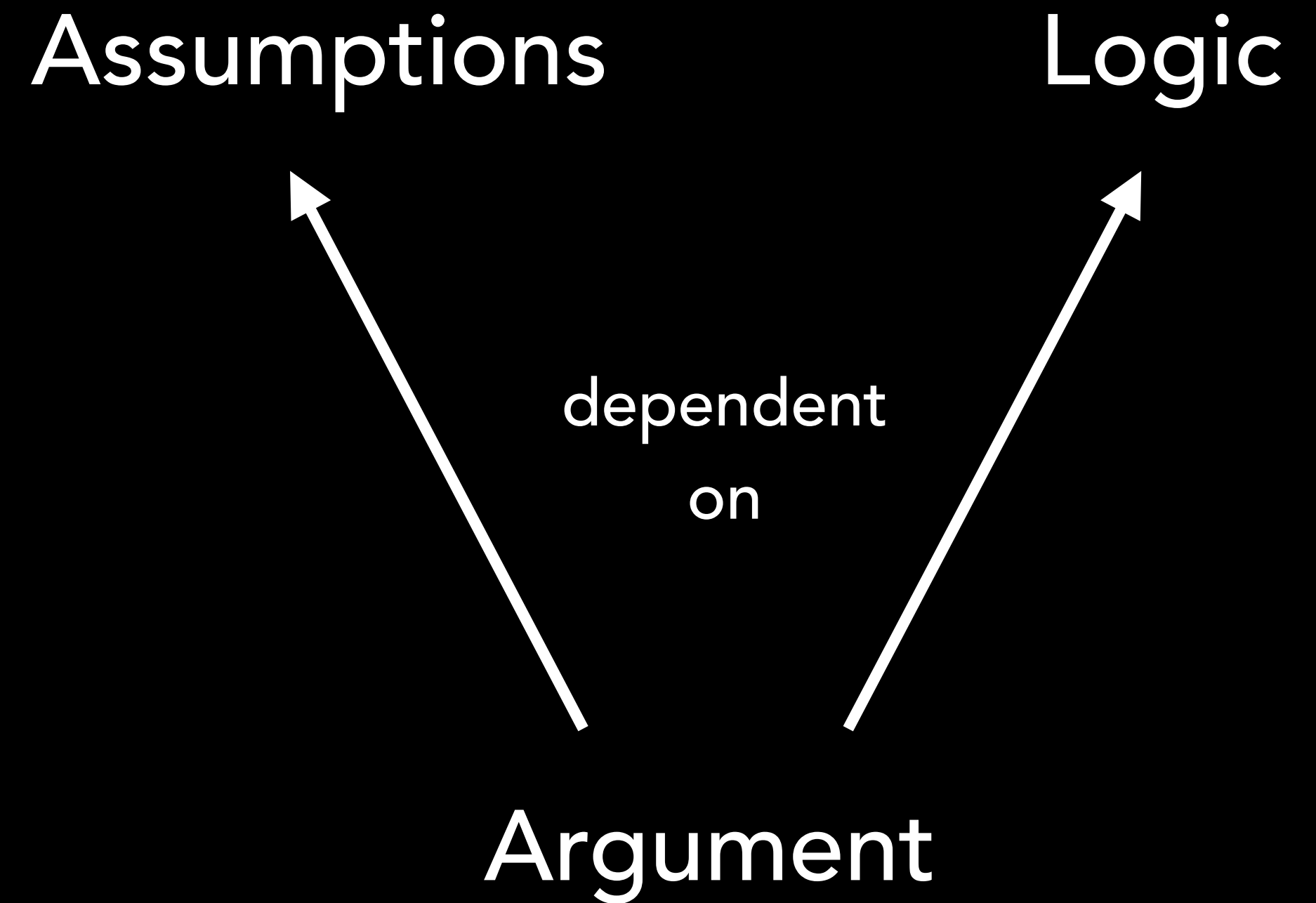
Chris Matyszek, c|net

Important Clarification

No!

We haven't proven the
existence of God using a
computer!

We have formally analysed a
prominent argument for the
existence of God using a
computer.



'Exact' science is possible ...



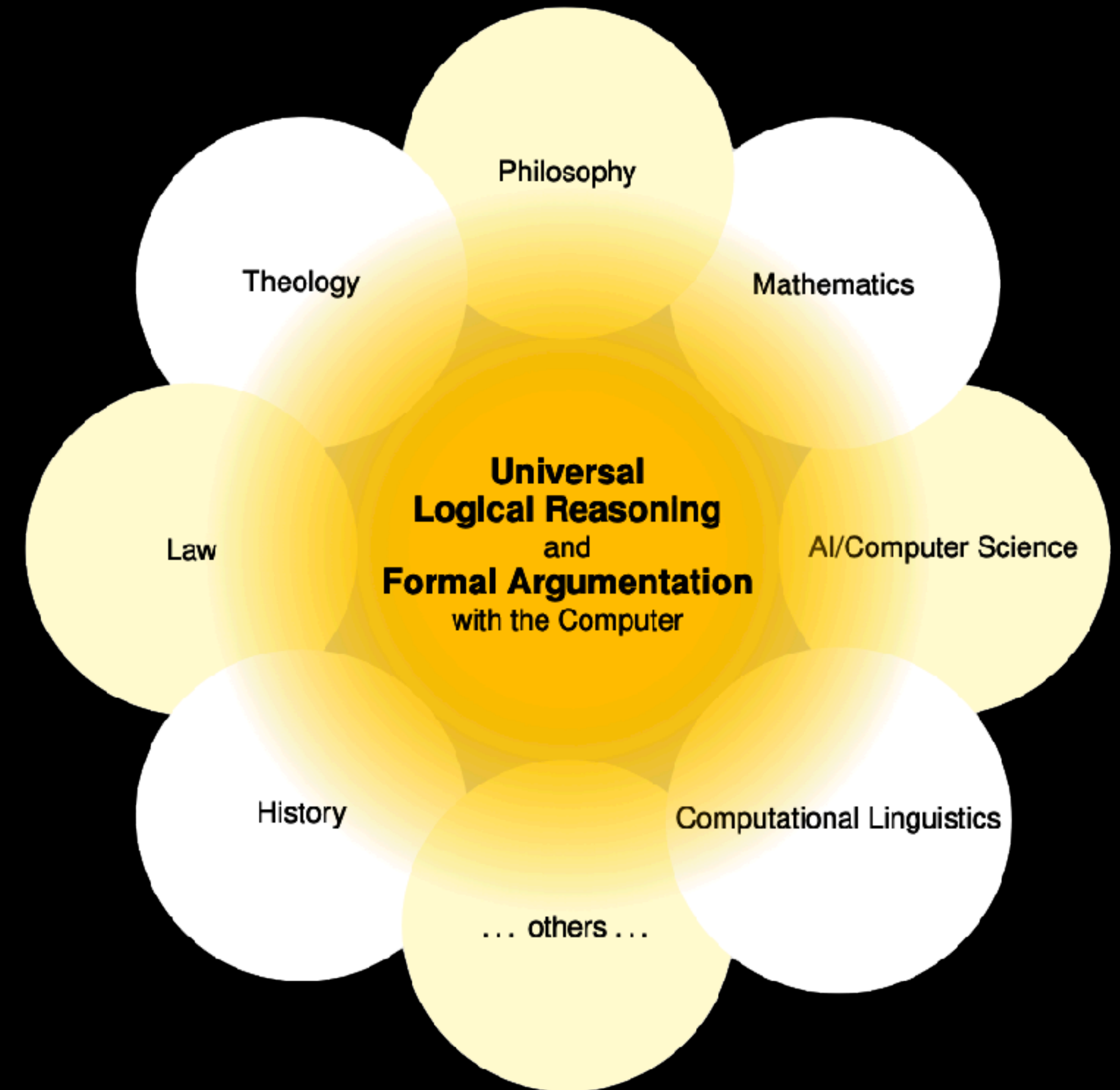
Es gibt systematische Methoden zur Lösung aller Probleme (auch Kunst etc.).

Es gibt eine wissenschaftliche (exakte) Philosophie {und Theologie}, welche die Begriffe der höchsten Abstraktheit behandeln.

Kurt Gödel: Maximen V

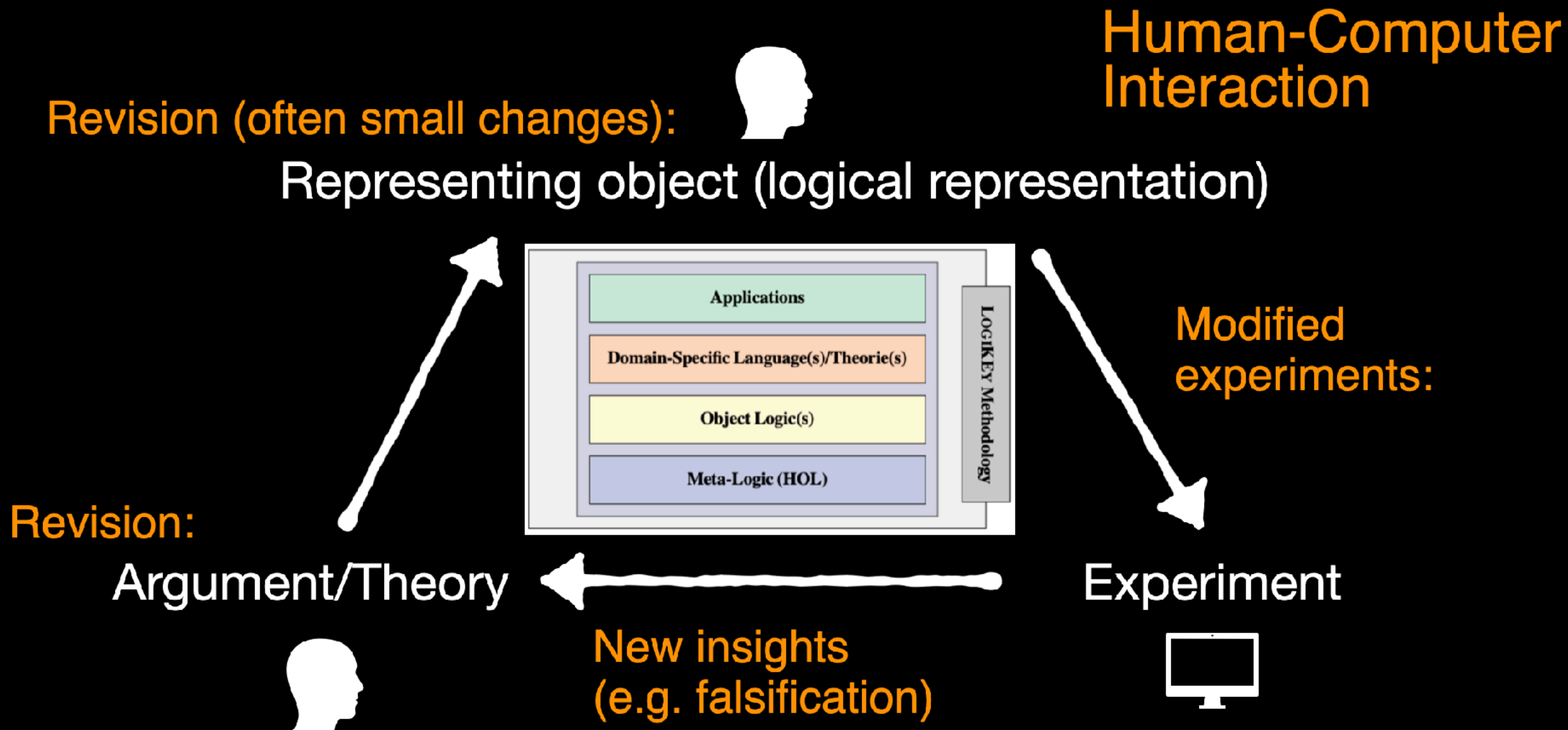


Kurt Gödel (1906-78)



... far beyond the realm of mathematics

LogiKey: Exploration Process



LogiKEy can be used with many different tools

Isabelle/HOL: Still my favorite tool for logic education and research

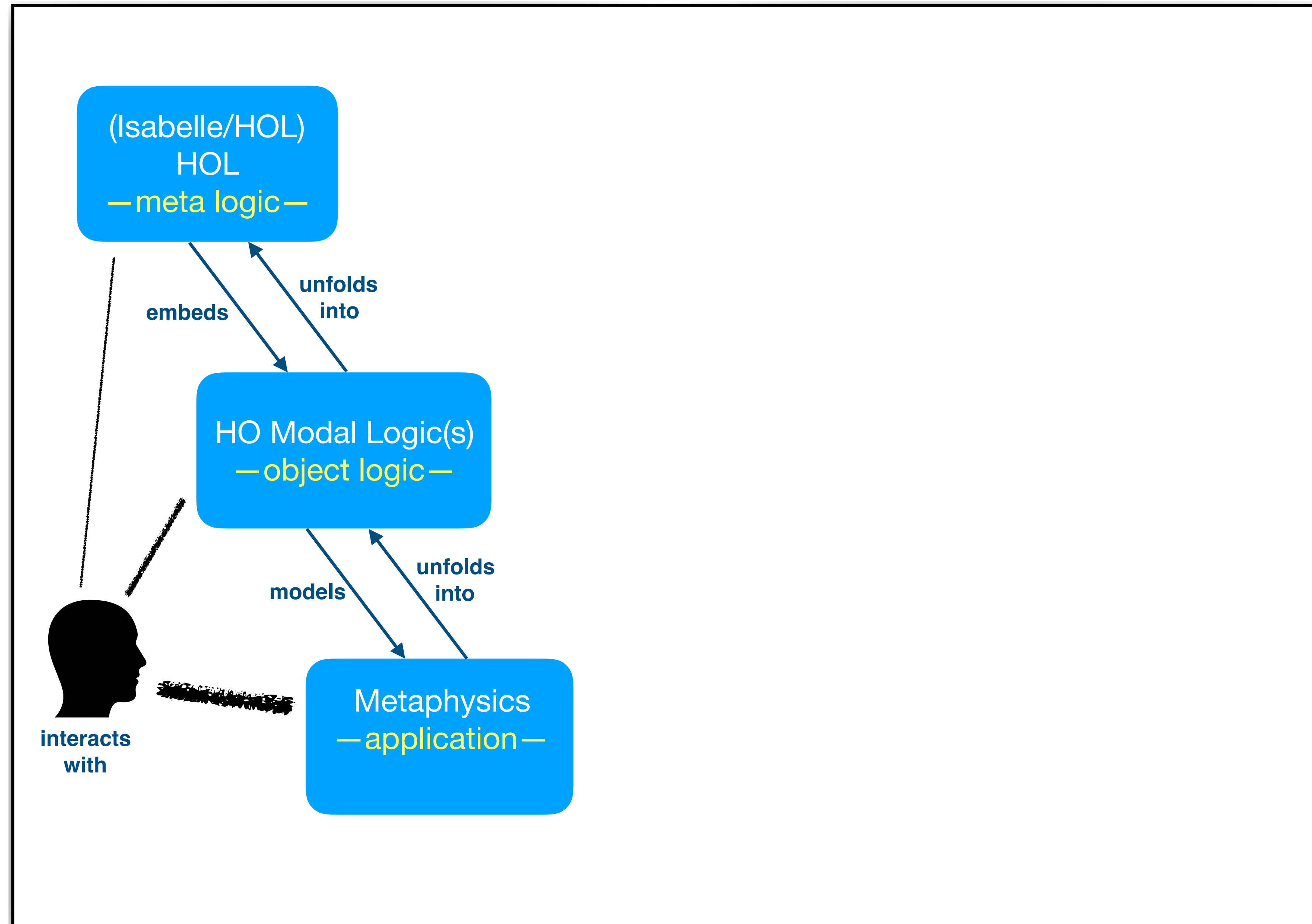
- Proof automation with **Sledgehammer** (FO & HO ATPs, SMT solvers)
- Model- and countermodel-finding with **Nitpick**
- Powerful **User Interface** (but this is a bottleneck at the same time)

Many other provers can be used instead (and we have demonstrated this):

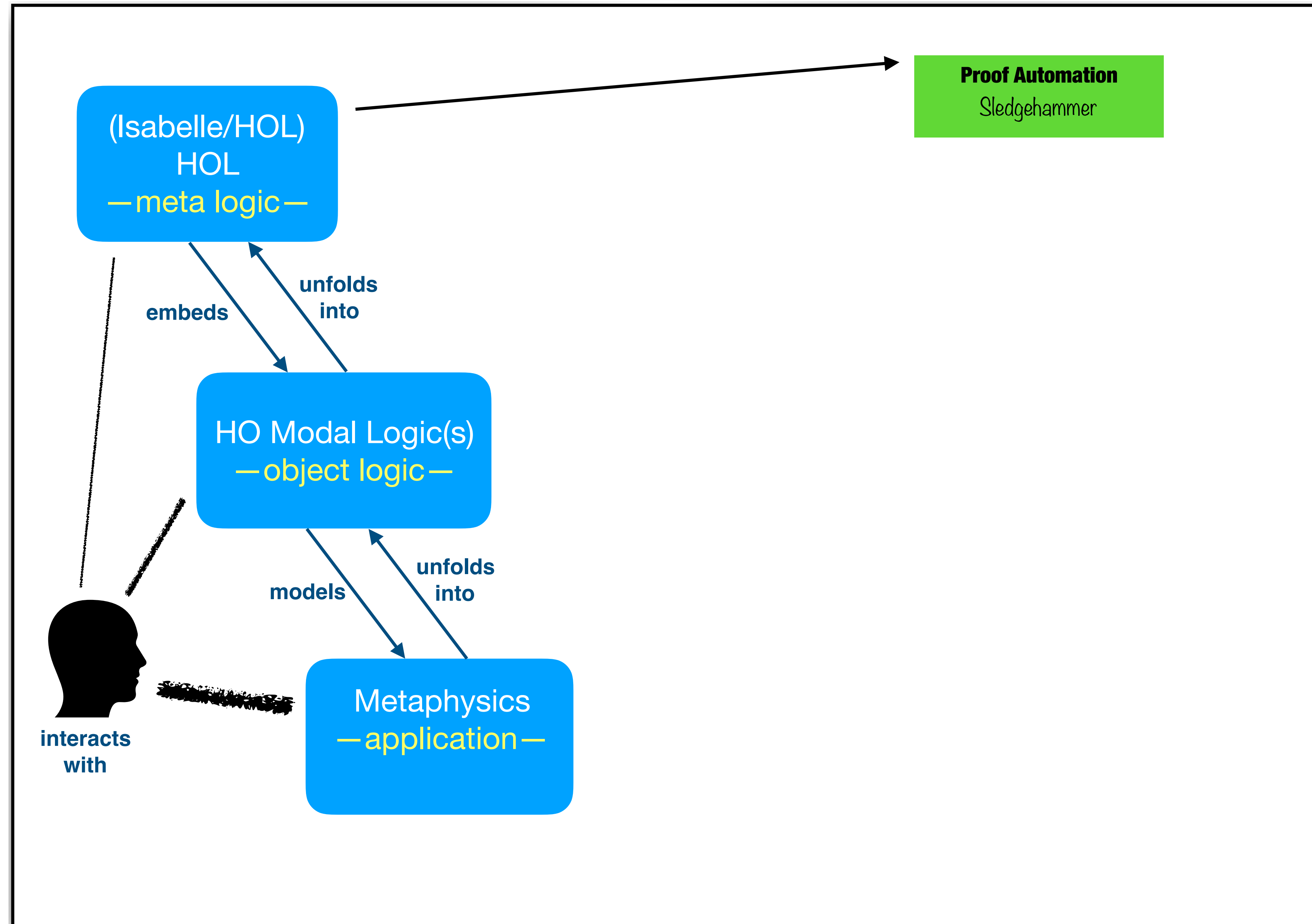
- Other ITPs: **Lean, HOL4, Coq, Agda, PVS, ...**
- Own HO ATPs: **LEO-II, LEO-III**
- Other HO-ATPs: **Vampire, E, Zipperposition, CVC5, ...**

Utilizing the LogiKEy Methodology within the Isabelle/HOL proof assistant system

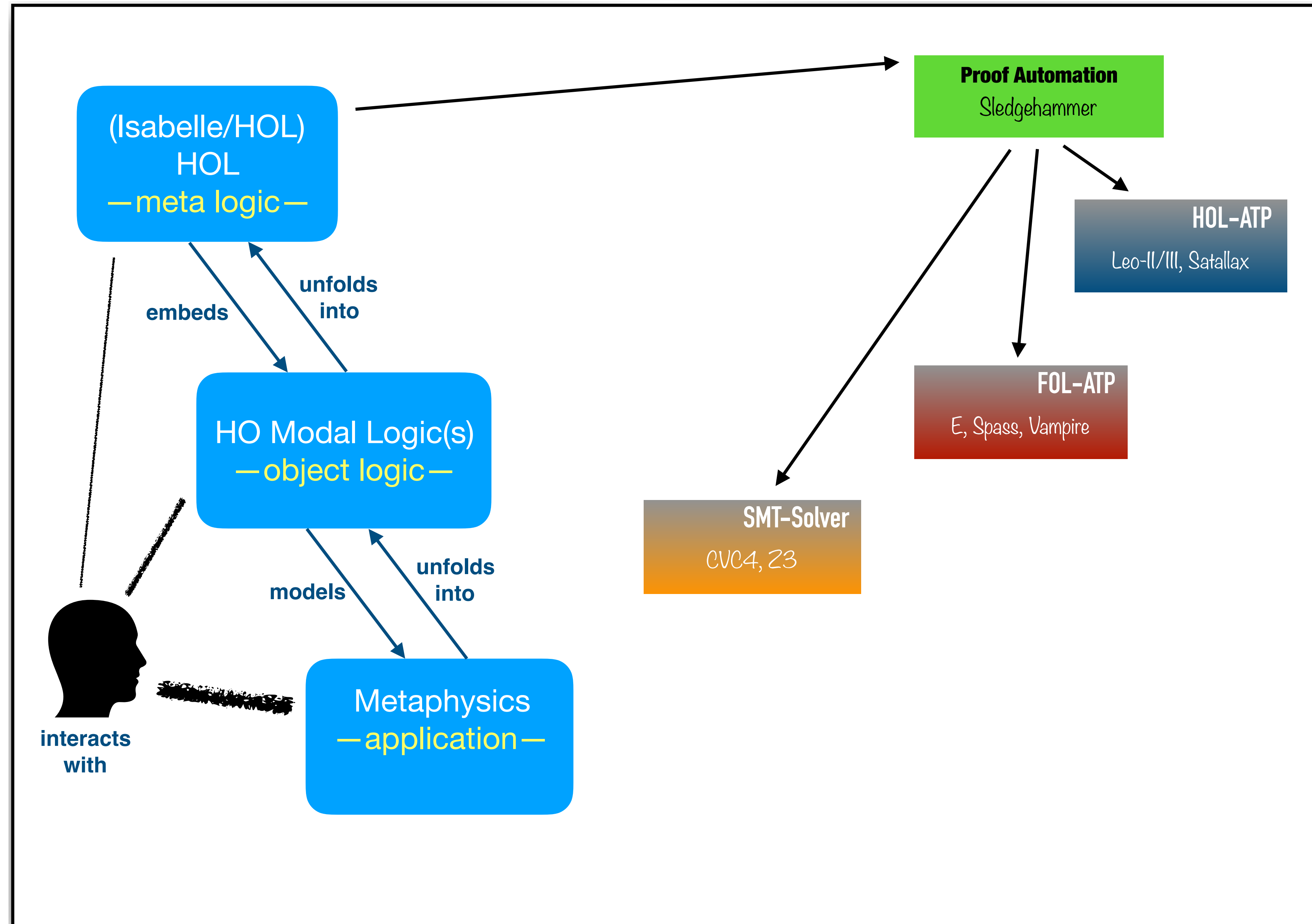
Universal Logical Reasoning in HOL



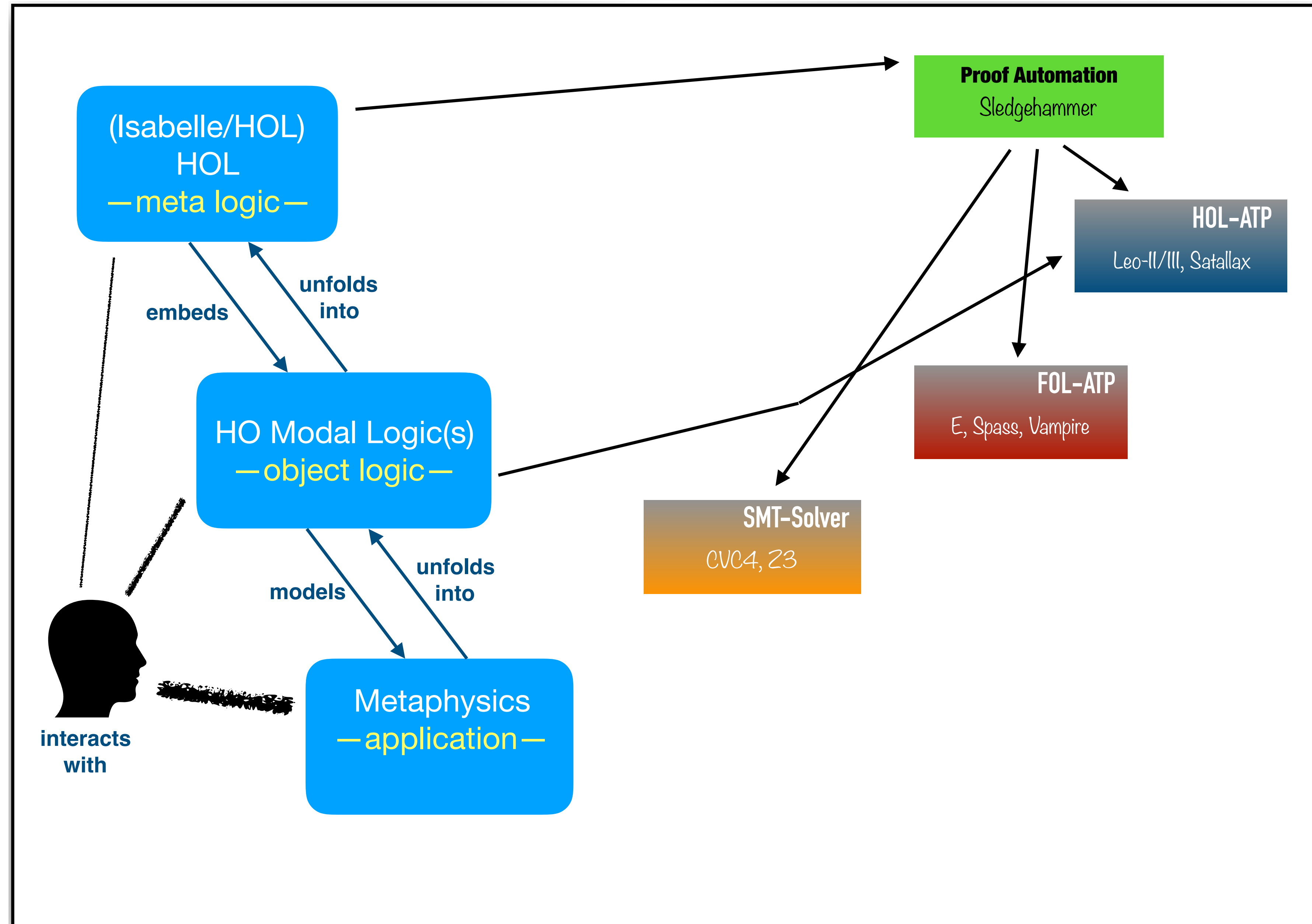
Universal Logical Reasoning in HOL



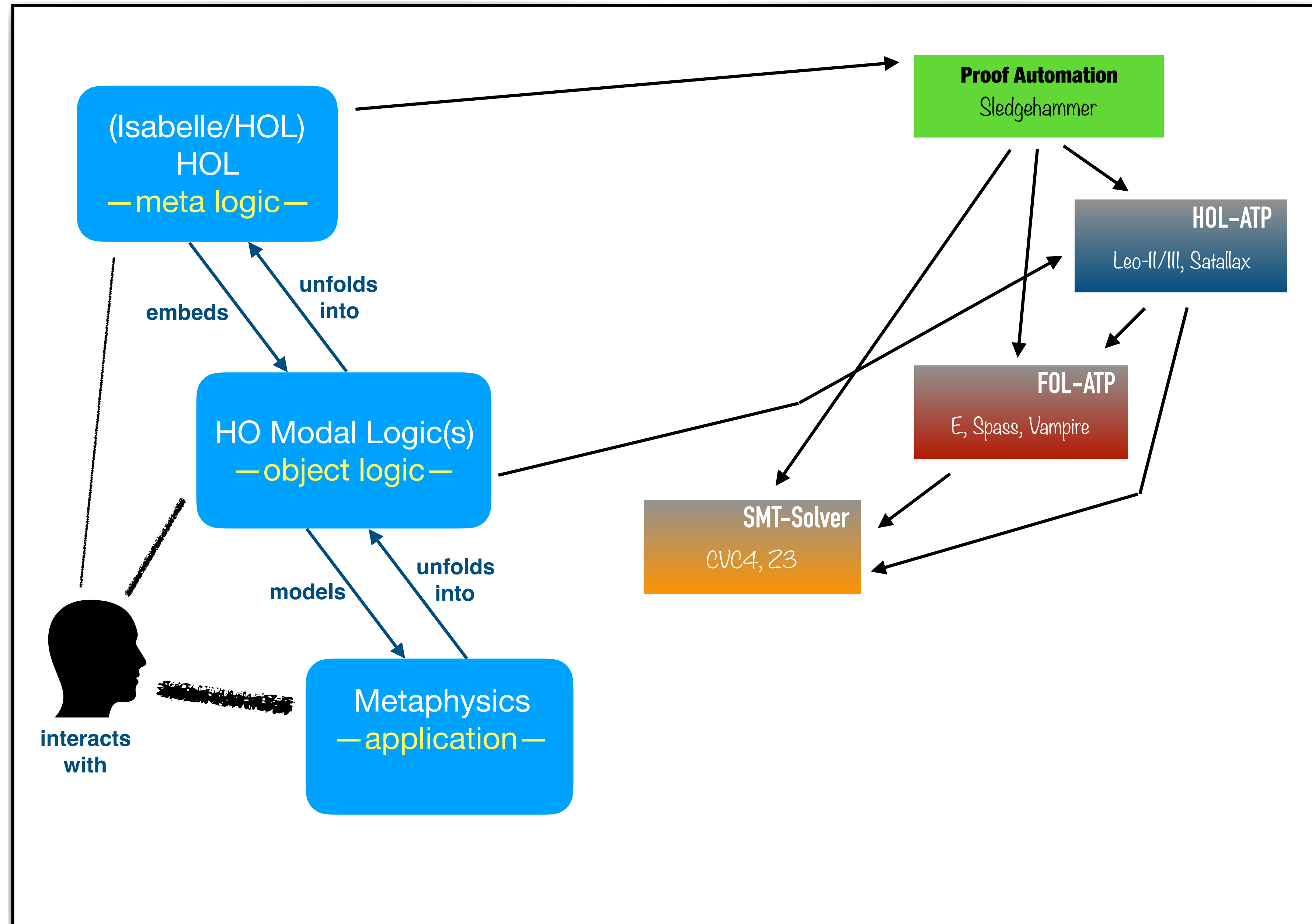
Universal Logical Reasoning in HOL



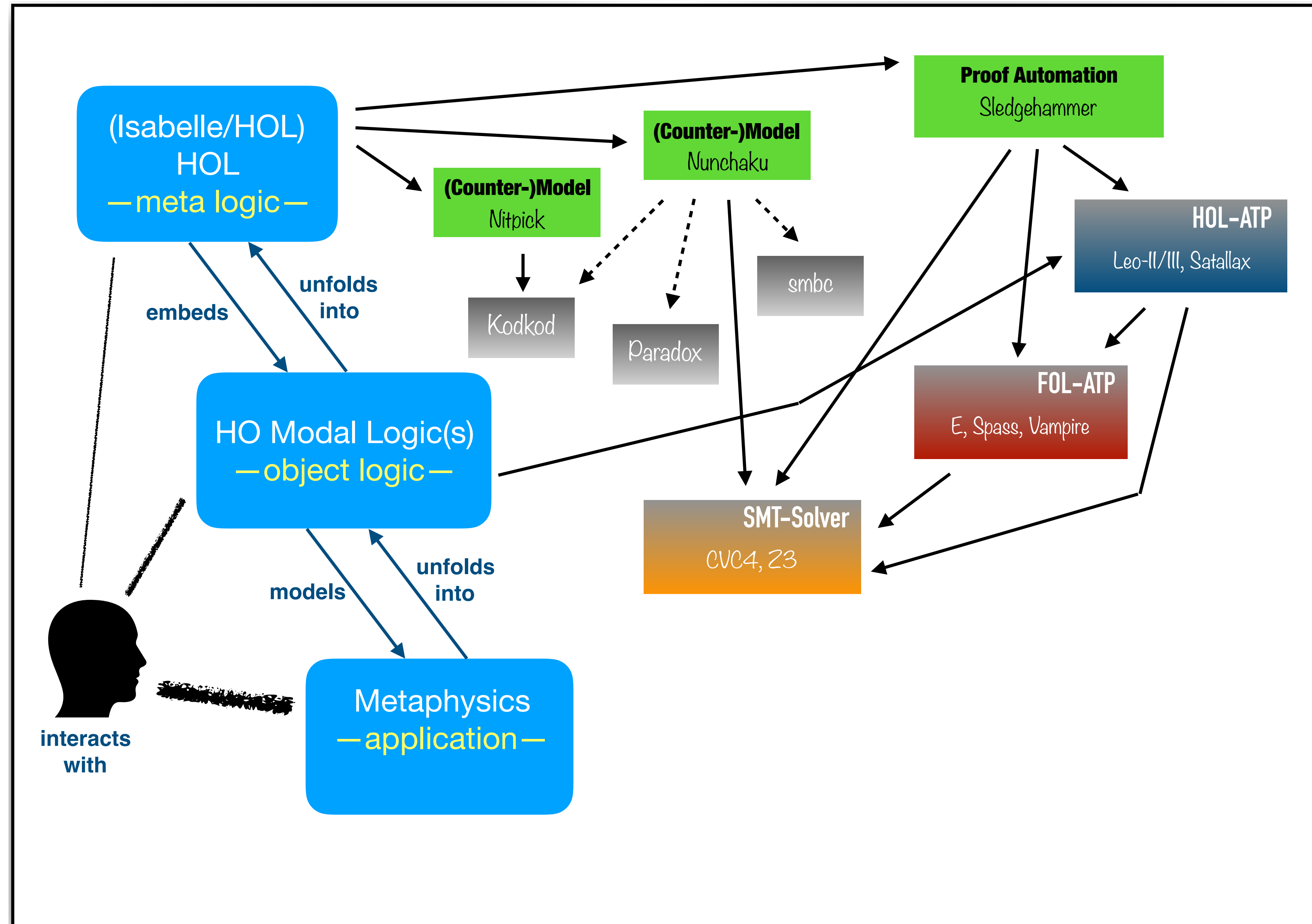
Universal Logical Reasoning in HOL



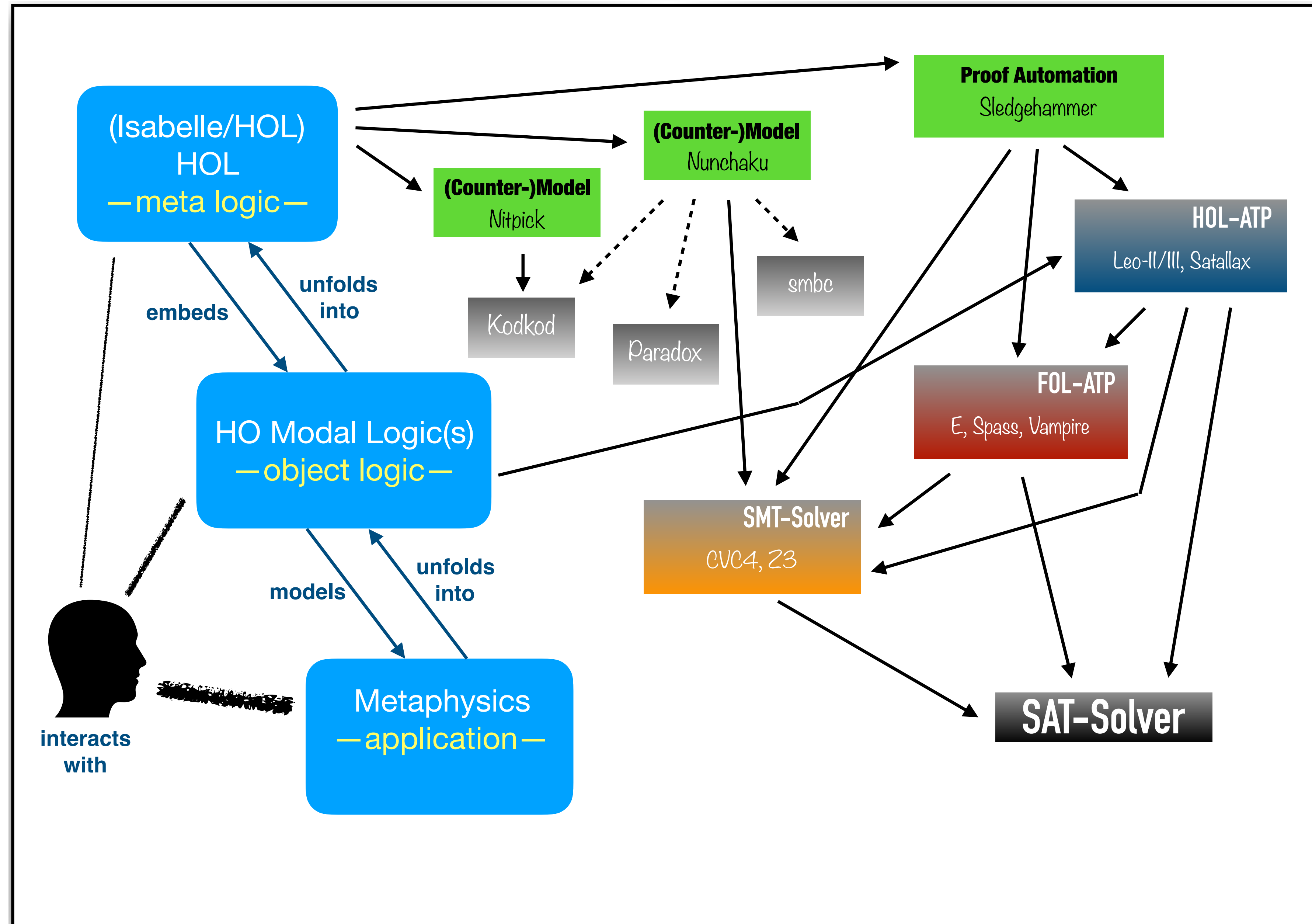
Universal Logical Reasoning in HOL



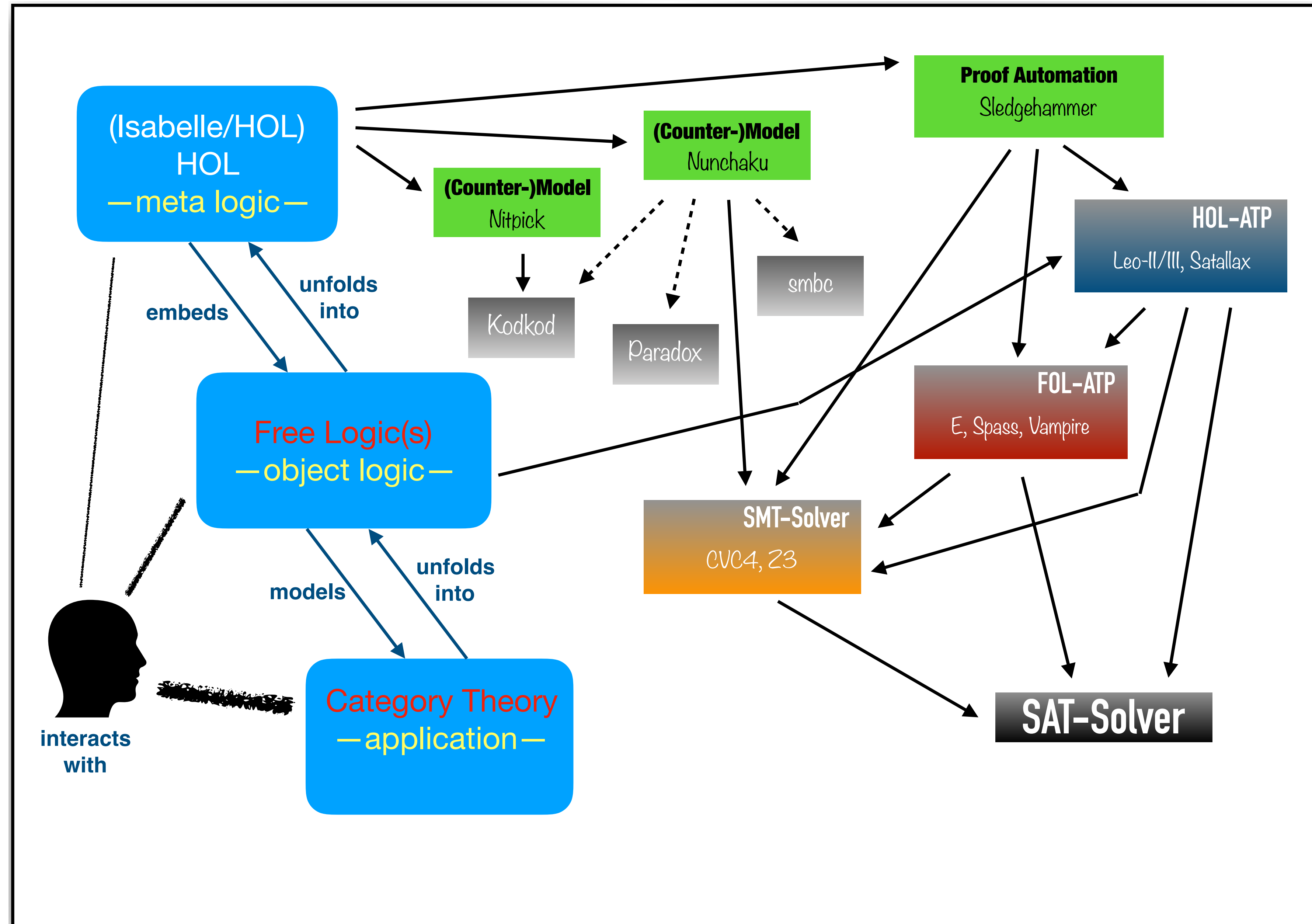
Universal Logical Reasoning in HOL



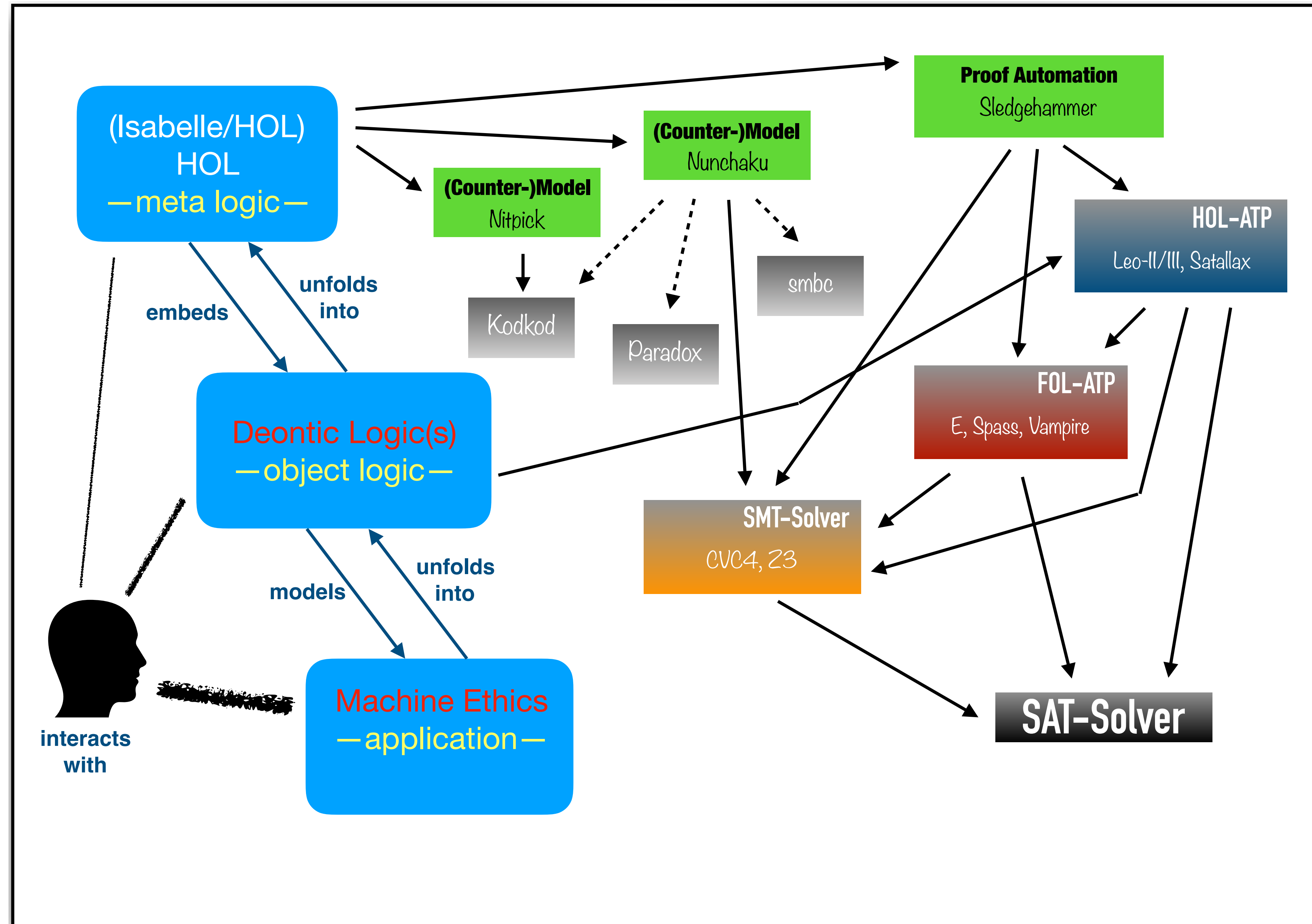
Universal Logical Reasoning in HOL



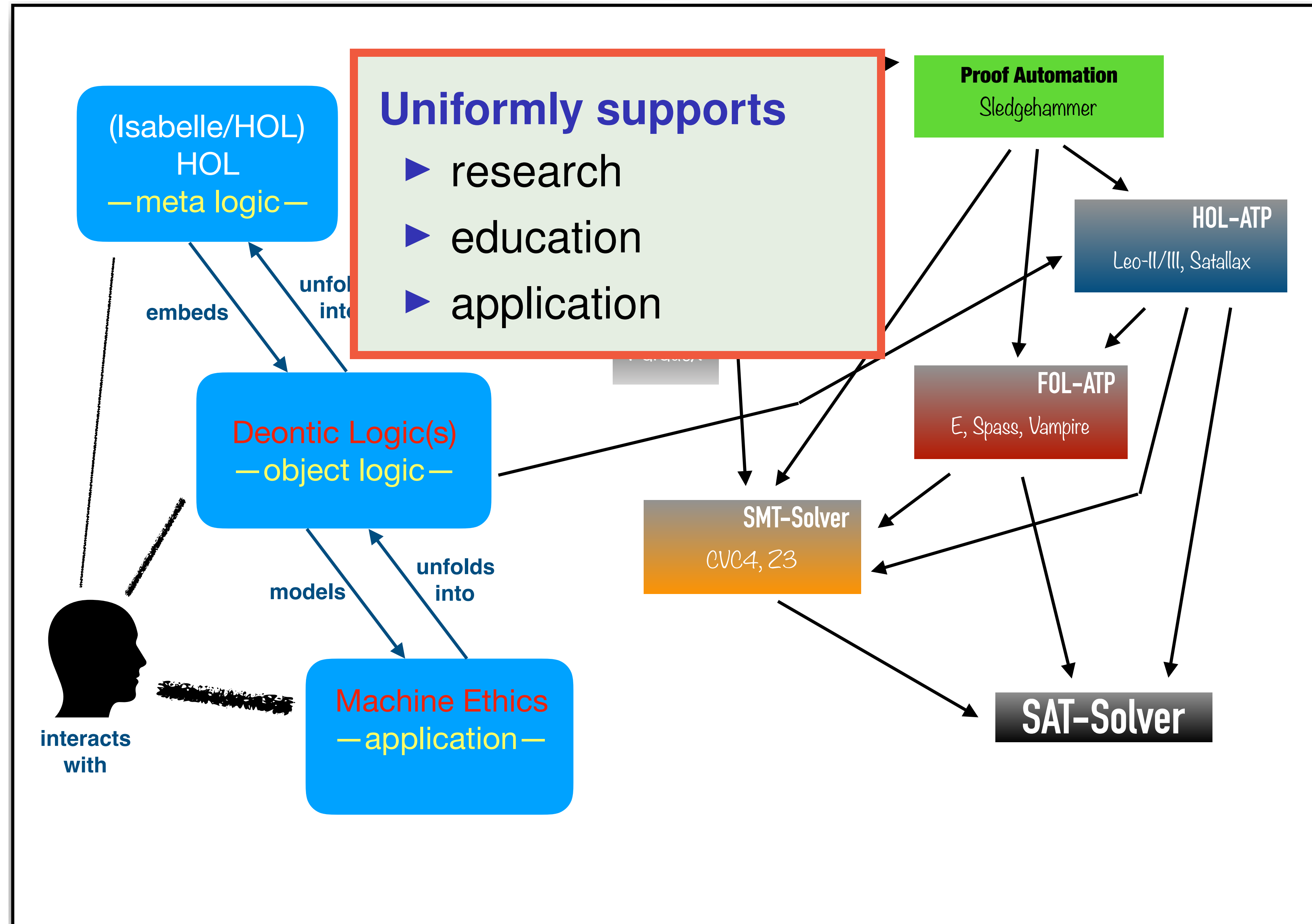
Universal Logical Reasoning in HOL



Universal Logical Reasoning in HOL



Universal Logical Reasoning in HOL



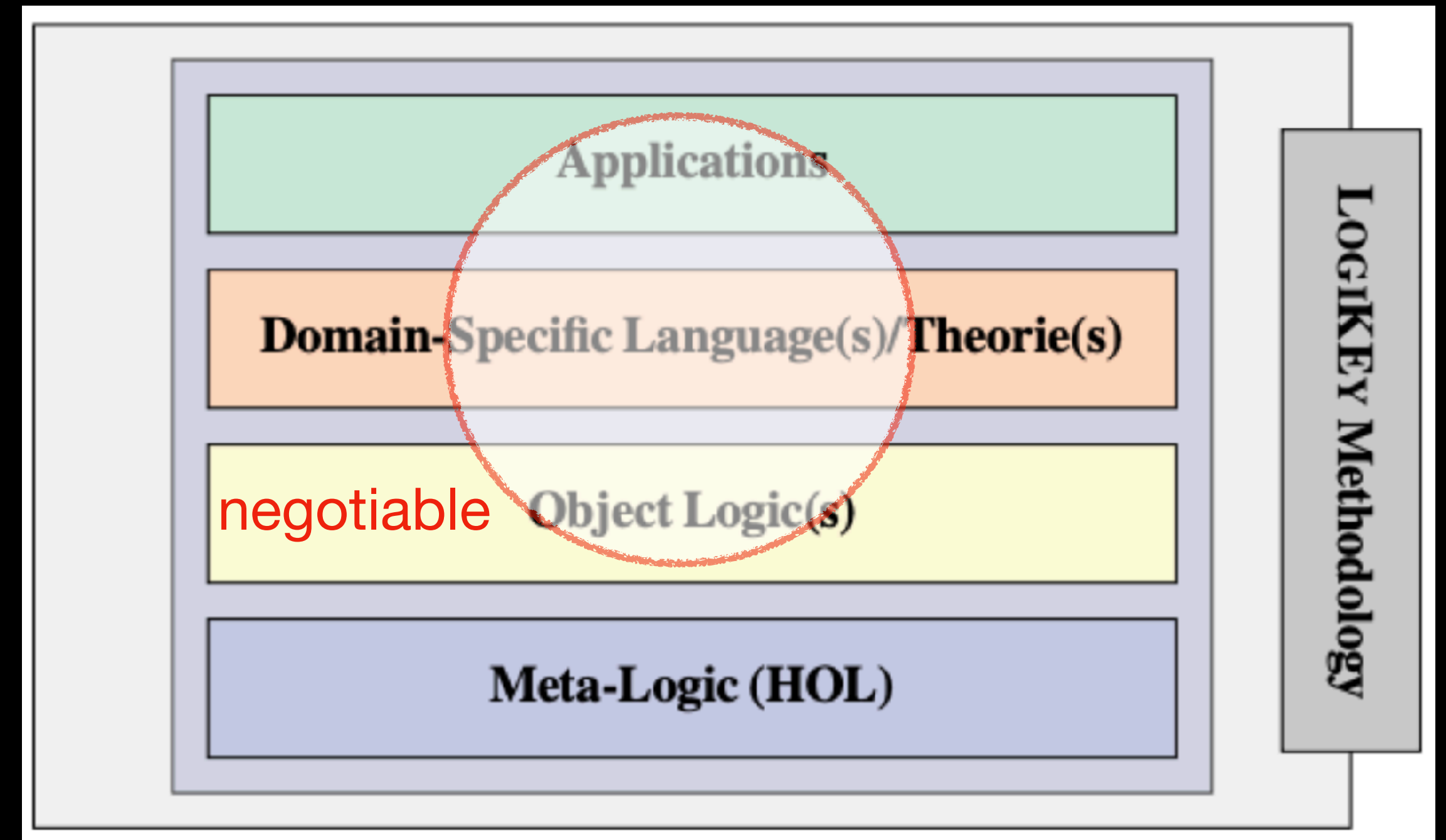
Summary: Logico-Pluralistic LogiKEy Approach

LogiKEy successfully applied for

- a wide range of object logics
- various object logic combinations
- different application domains

LogiKEy in Isabelle/HOL

- good proof automation
- even more valuable: model finding
- good syntax representations



LogiKEy offers a uniform methodology and infrastructure where even object logics and their combinations become negotiable and objects of study.