ESSLLI 2026

Experimenting with the LogiKEy Framework & Methodology

Day 3: Normative Reasoning

Christoph Benzmüller and Luca Pasetto

Course Schedule - Normative Reasoning

- **Today:** Overview, Standard Deontic Logic, beyond Standard Deontic Logic, Preferences

- **Tomorrow:** Agency and Legal Relations, Normative Positions, Epistemic Rights

The DEON community

https://deonconferences.org

DeonticLogic.org

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  - Society of Deontic Logic and Normative Systems
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Society of Deontic Logic and Normative Systems

The **Society of Deontic Logic and Normative Systems** is concerned with fostering research and communication on deontic logic and normative systems, and aims to build bridges between disciplines like ethics, law, linguistics, philosophy, computer science, game theory, the social sciences, artificial intelligence, and mathematics.

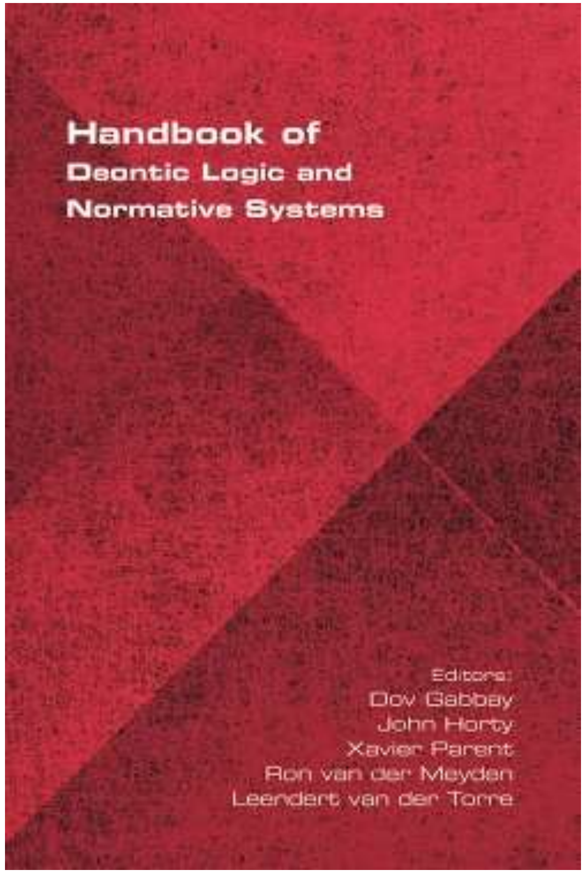
Its primary activities include organizing a series of prestigous conferences, the International Conferences on Deontic Logic and Normative Systems (DEON), cooperating with workshops on related topics, maintaining informational materials at this website, and maintaining a DEON-related mailing list.

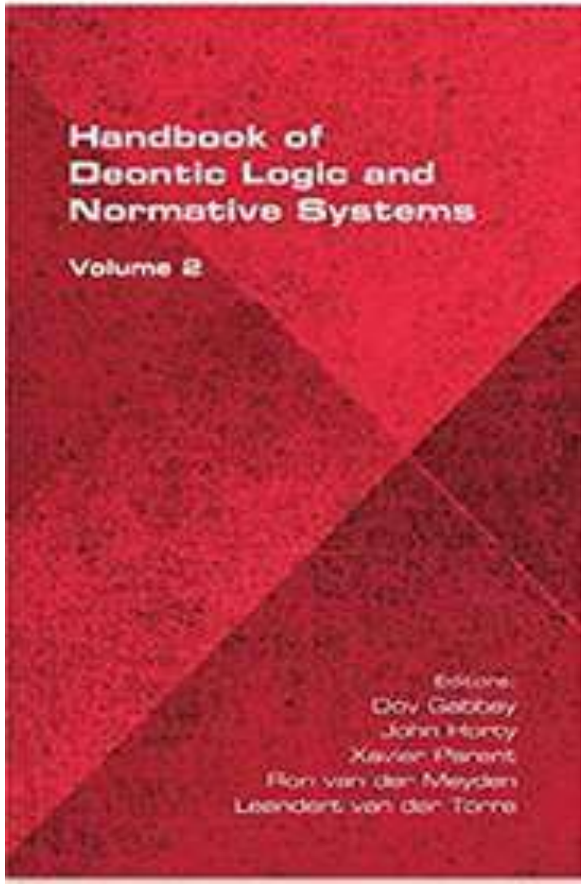
The DEON conferences series has been held biannually since 1991. While its initiators where mostly from computer science, the series now brings together researchers from various fields, including computer science, philosophy, legal theory, and linguistics.

> **DEON 2025** will be held on 30.06.-03.07.2025 at TU Wien, Austria. The Call for Papers is open!

Handbooks

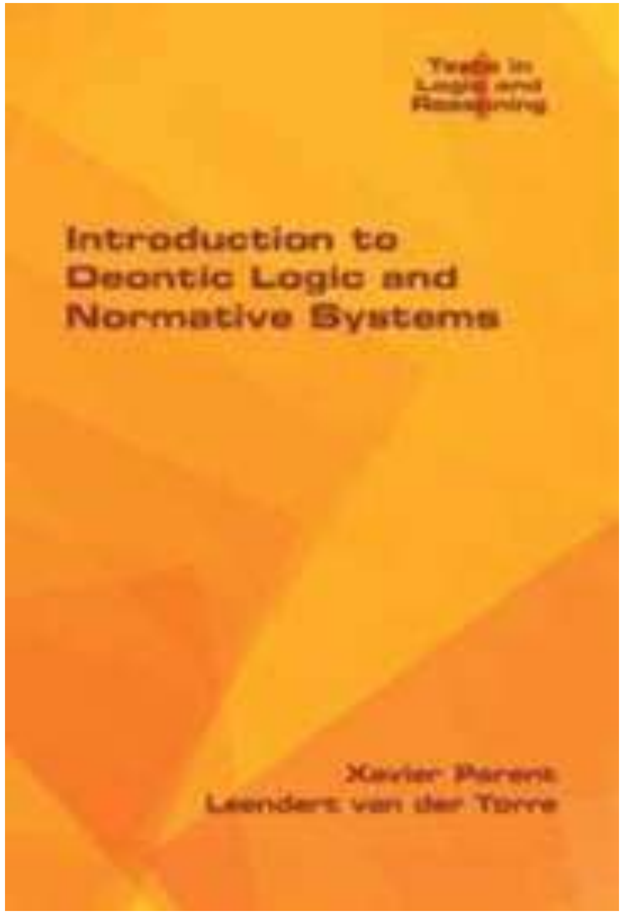
http://collegepublications.co.uk





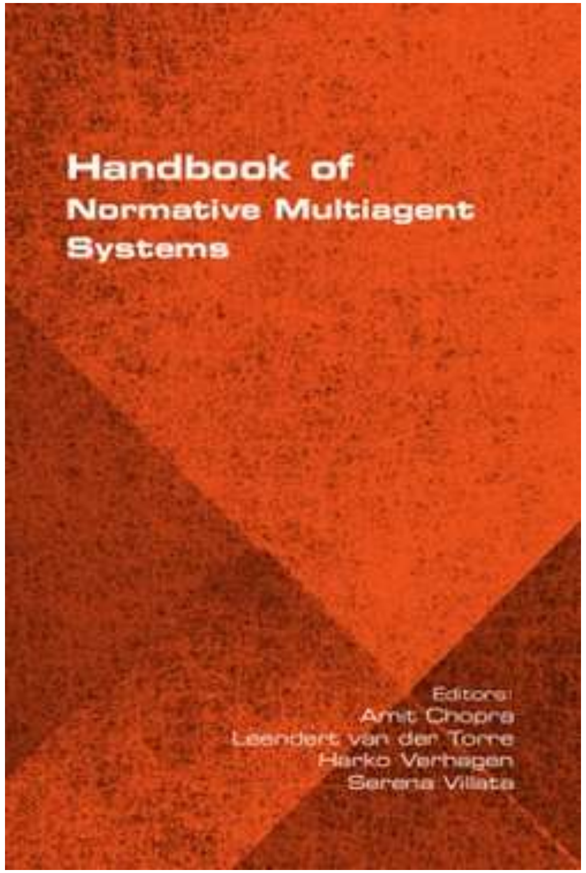
Textbook

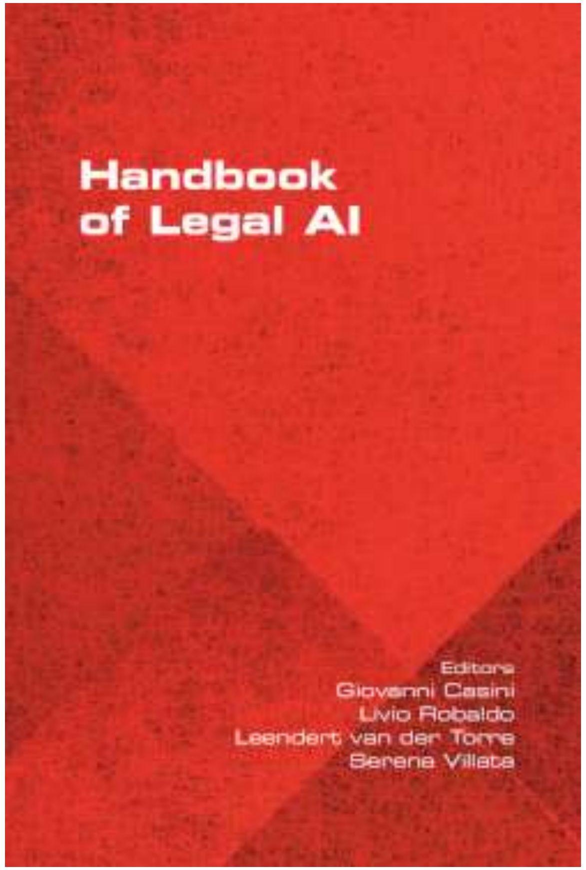
http://collegepublications.co.uk



Other Handbooks

http://collegepublications.co.uk





Normative Reasoning: Philosophical challenges and foundations

Based on slides by Réka Markovich

Normative Reasoning

Normative Reasoning

reasoning about

what should be the case,
what should be done,
what you should, shouldn’t or are permitted to do

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If you make a promise, keep it.
You made a promise.

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Civil Code:
“any person who causes damage to another person wrongfully has to pay for it”

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Premise 1	if *x* causes damage wrongfully to *y*, *x* has to pay for it (to *y*)
Premise 2	*a* caused damaged wrongfully to *b*
Conclusion	*a* has to pay for it (to *b*)

Jørgensen dilemma (1937)

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Keep it.

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It shouldn't be working, but it works!

The big issue with Deontic Logic

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a norm is not true or false
a norm prescribes, not describes

The big issue with Deontic Logic

a norm is not true or false
a norm prescribes, not describes

…even if the statement is descriptive:

What makes the sentence ‘*p should be the case*’ true?

Founding Deontic Logic

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Georg Henrik von Wright

Deontic Logic, *Mind*, 1951

Founding Deontic Logic

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uses deontic operators prefixed act-names (action types) A, B, …

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PA is to be understood as performing A is permitted

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PA is to be understood as performing A is permitted

OA $\Leftrightarrow\_def \neg P \neg A$

FA $\Leftrightarrow\_def \neg PA$

IA $\Leftrightarrow\_def\ PA \wedge P\neg A$

Deontic Logic, *Mind*, 1951

Deontic Logic as Modal Logic

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Gottfried Wilhelm Leibniz:
Elementa juris naturalis (1672)

Deontic Logic as Modal Logic

Gottfried Wilhelm Leibniz:
Elementa juris naturalis (1672)

Iuris Modalia (legal modalities)

debitum	obligatory
licitum	permitted
illicitum	forbidden
indifferentum	facultative

analogies between alethic and deontic modalities

Founding Deontic Logic

Georg Henrik von Wright

Principle of Deontic Distribution: $P(A \vee B) \Leftrightarrow PA \vee PB$

Principle of Permission: $PA \vee P\neg A$

Rule of Extensionality: If A and B logically equivalent, then PA and PB are equivalent too

Principle of Deontic Contingency: $O(A \vee \neg A)$ and $\neg P(A \wedge \neg A)$ are not theorems

Deontic Logic, *Mind*, 1951

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(no semantics at this time)

Deontic Logic, *Mind*, 1951

Monadic Deontic Logic

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alterations on von Wright’s system in Deontic Logic (bringing it closer to modal logic)

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propositions instead of act-names

adding the necessitation rule

Monadic Deontic Logic

Modal system **D** (or **KD**)

Monadic Deontic Logic

Modal system **D** (or **KD**)

Language

Definition 1. *Let $\mathbb{P}$ be a set of atomic propositions. The language $\mathcal{L}$ of monadic deontic logic is generated by the following BNF (Backus Normal Form):*

$$\phi ::= p \mid \neg\phi \mid (\phi \land \phi) \mid \bigcirc\phi$$

where $p \in \mathbb{P}$.

Monadic Deontic Logic

Modal system **D** (or **KD**)

further connectives and operators are introduced with the following definitions:

Monadic Deontic Logic

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disjunction	$\phi \vee \psi$	*is*	$\neg(\neg\phi \wedge \neg\psi)$
implication	$\phi \rightarrow \psi$	*is*	$(\neg\phi) \vee \psi$
equivalence	$\phi \leftrightarrow \psi$	*is*	$(\phi \rightarrow \psi) \wedge (\psi \rightarrow \phi)$
verum	$\top$	*is*	$p \vee \neg p$ *(for some* $p \in \mathbb{P}$*)*
falsum	$\bot$	*is*	$\neg\top$
permission	$P\phi$	*is*	$\neg \bigcirc \neg\phi$
prohibition	$F\phi$	*is*	$\bigcirc\neg\phi$

Monadic Deontic Logic

Modal system **D** (or **KD**)

Proof Theory

Monadic Deontic Logic

Modal system **D** (or **KD**)

Proof Theory

Definition 2. D *is the proof system consisting of the following axiom schemas and rule schemas.*

$\vdash \phi$, *where* ϕ *is a propositional tautology*	(PL)
$\vdash \bigcirc(\phi \to \psi) \to (\bigcirc\phi \to \bigcirc\psi)$	($\bigcirc$-K)
$\vdash \bigcirc\phi \to P\phi$	($\bigcirc$-D)
If $\vdash \phi$ *and* $\vdash \phi \to \psi$ *then* $\vdash \psi$	(MP)
If $\vdash \phi$ *then* $\vdash \bigcirc\phi$	($\bigcirc$-Nec)

Monadic Deontic Logic

Modal system **D** (or **KD**)

Semantics: relational and based on ideality

Monadic Deontic Logic

Modal system **D** (or **KD**)

Semantics: relational and based on ideality

Definition 3 (Relational model)**.** *A relational model* M *is a tuple*

$$(W,\ R,\ V)$$

where:

W *is a (non-empty) set of states (also called "possible worlds")*
$s, t, \ldots$ W *is called the universe of the model.*

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W is a (non-empty) set of states (also called "possible worlds") $s, t, \ldots$ W is called the universe of the model.
$R \subseteq W \times W$ is a binary relation over W. It is understood as a relation of deontic alternativeness: sRt (or, alternatively, $(s, t) \in R$) says that t is an ideal alternative to s, or that t is a "good" successor of s. The first one is "good" in the sense that it complies with all the obligations true in the second one.

Monadic Deontic Logic

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Furthermore, R is subject to the following constraint:

$$(\forall s \in W)(\exists t \in W)(sRt) \qquad \text{(seriality)}$$

This means that the model does not have a dead end, a state with no good successor.

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$V : \mathbb{P} \mapsto 2^W$ *is a valuation function assigning to each atom p a set $V(p) \subseteq W$ (intuitively the set of states at which p is true).*

Monadic Deontic Logic

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Definition 4 (Satisfaction). *Given a relational model $M = (W, R, V)$ and a state $s \in W$, we define the satisfaction relation $M, s \models \phi$ (read as "state s satisfies ϕ in model M", or as " s makes ϕ true M") by induction on the structure of ϕ using the following clauses:*

- $M, s \models p$ *iff* $s \in V(p)$
- $M, s \models \neg\phi$ *iff* $M, s \not\models \phi$
- $M, s \models \phi \wedge \psi$ *iff* $M, s \models \phi$ *and* $M, s \models \psi$
- $M, s \models \bigcirc\phi$ *iff for all* $t \in W$, *if* sRt *then* $M, t \models \phi$

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- $M, s \vDash \bigcirc\phi$ *iff for all* $t \in W$, *if* sRt *then* $M, t \vDash \phi$

Definition 5 (Validity). *A formula ϕ is valid (notation: $\vDash \phi$) whenever, for all relational models $M = (W, R, V)$ and all states $s \in W$, $s \vDash \phi$.*

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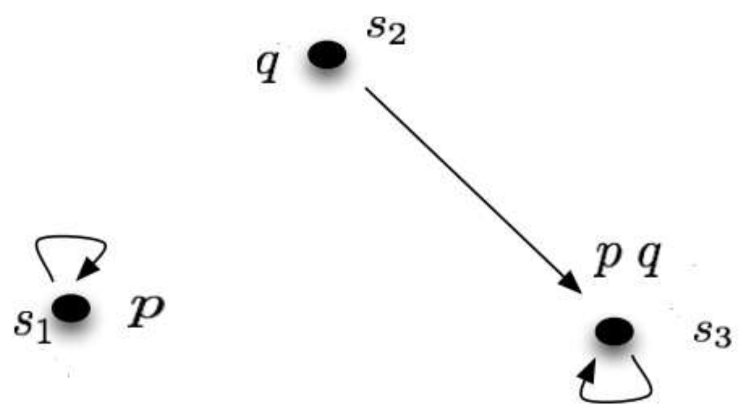

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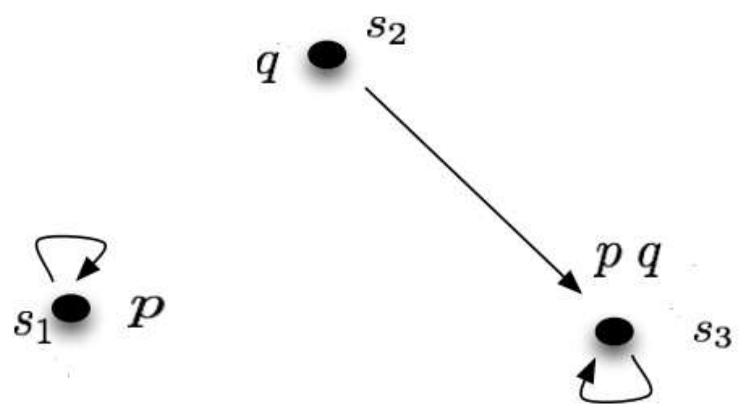

- $s_1 \models p$ (since $s_1 \in V(p)$)
- $s_1 \models \neg q$ (since $s_1 \not\models q$)
- $s_1 \models p \wedge \neg q$
- $s_1 \models \bigcirc(p \wedge \neg q)$ (s_1 has only one good successor: itself).

Alethic Modalities vs Deontic Modalities

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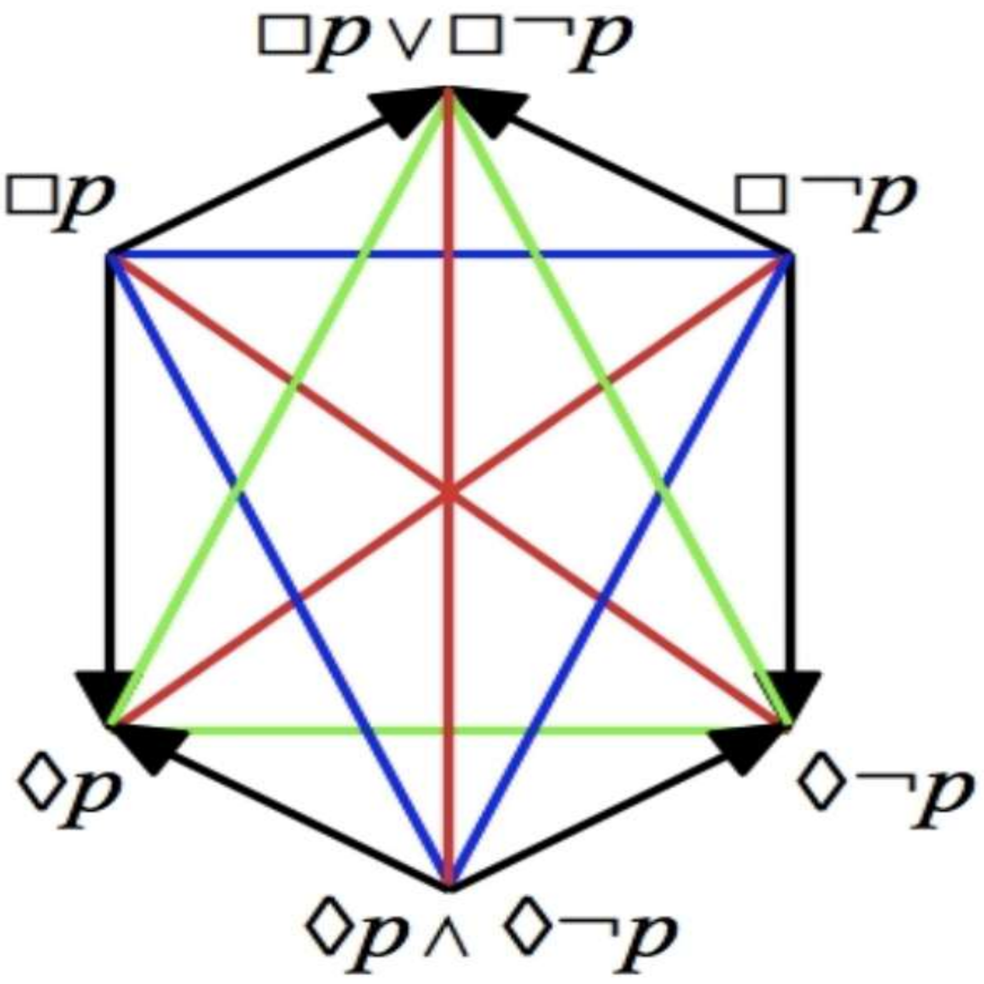
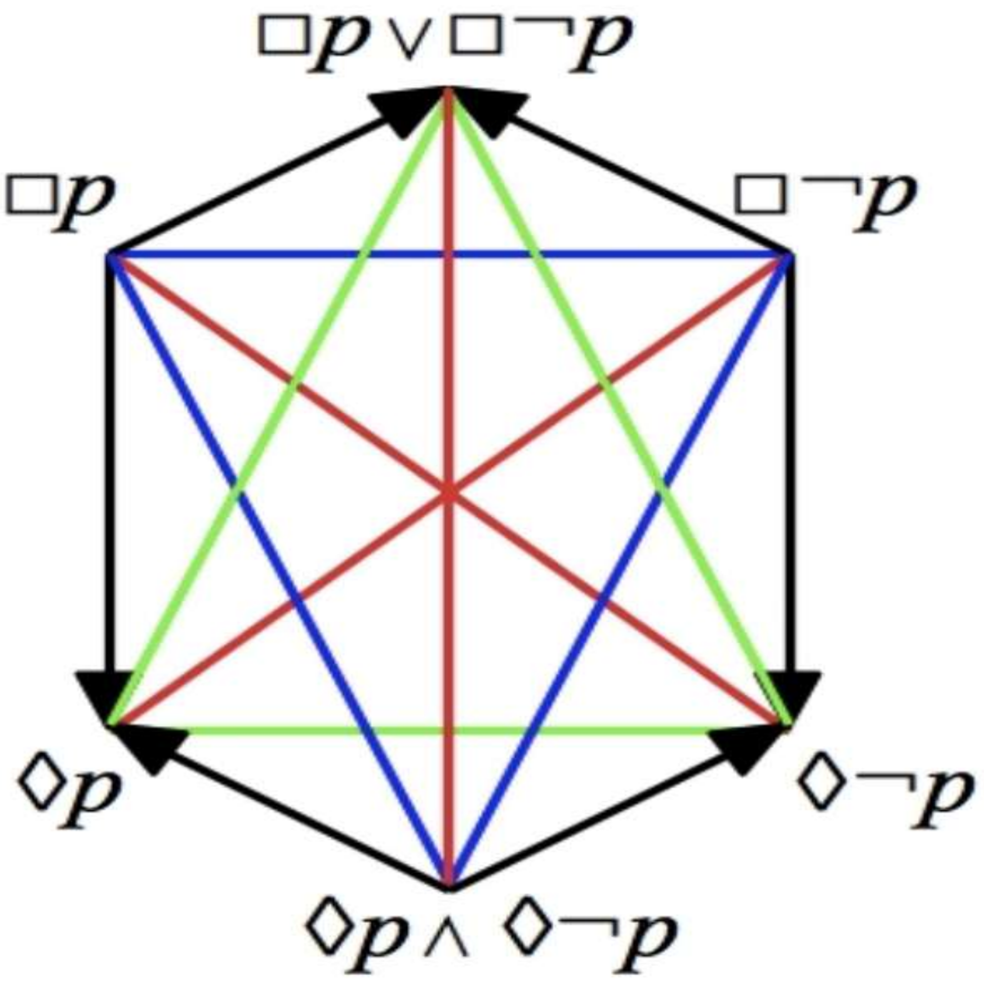


Alethic hexagon (Sesmat–Blanché)
(graphics: Smesseaert and Demey)

Alethic Modalities vs Deontic Modalities



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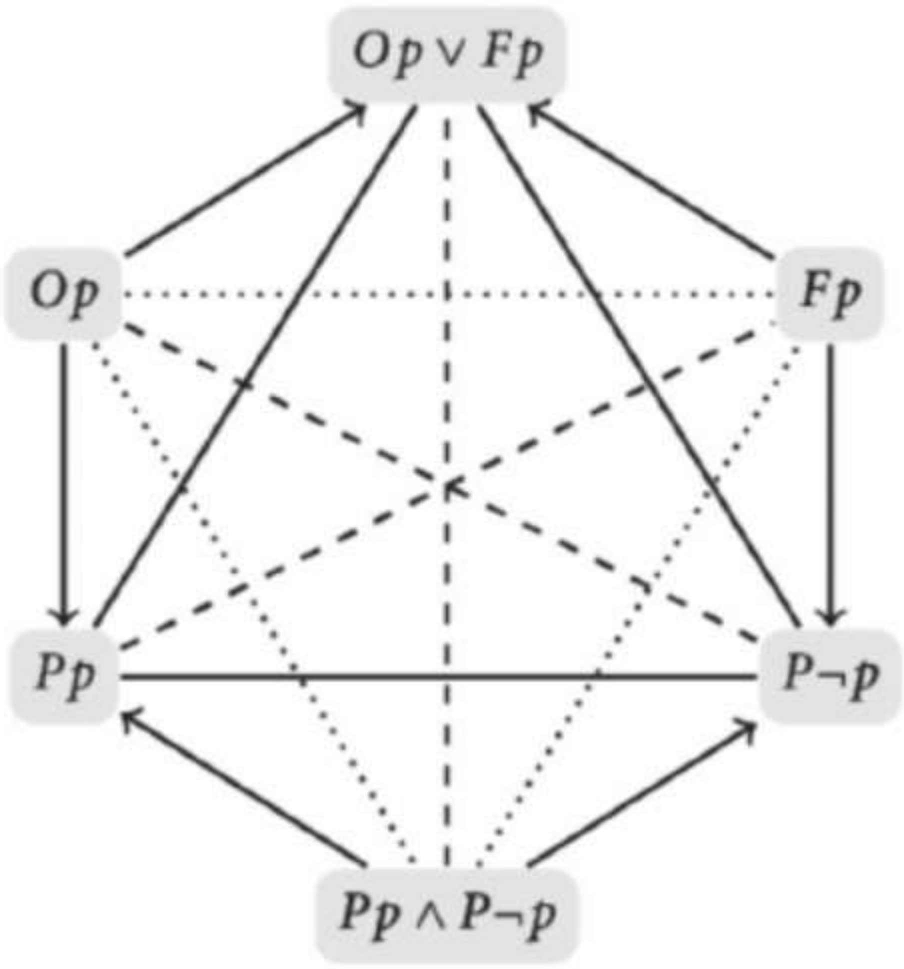

Deontic hexagon (Kalinowski, 1972)
(figure: Basic-Zarnic, 2014)

Alethic Modalities vs Deontic Modalities

most important differences in theorems

alethic	deontic
$\Box\varphi \rightarrow \varphi$	$O\varphi \rightarrow P\varphi$

Alethic Modalities vs Deontic Modalities

most important differences in theorems

alethic	deontic
$\Box\varphi \to \varphi$	$O\varphi \to P\varphi$
$\varphi \to \Diamond\varphi$	does not hold

Questions, Puzzles, Paradoxes

Deontic Modalities' Interdefinability

What's the difference between

a legal code permits something vs. doesn't say anything about it

Deontic Modalities' Interdefinability

What's the difference between

a legal code permits something	vs.	doesn't say anything about it
strong permission	vs.	weak permission

Free Choice Permission

You may have cake or ice cream
You may have cake and you may have ice cream

Free Choice Permission

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$$P(\mathrm{c} \vee i)$$

$$Pc \wedge Pi$$

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intuitive to say: $P(\mathrm{p} \vee q) \longrightarrow Pp \wedge Pq$

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But in **D**:

$if\ \ \vdash p \longrightarrow q\ then \vdash Op \longrightarrow Oq$ (derived derivation rule)

$Pp \longrightarrow P(p \vee q)$ (derived theorem)

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So, if we add, we would have: $Pp \longrightarrow Pp \wedge Pq$

$Pp \longrightarrow Pq$

if anything is permitted, then everything is permitted

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if anything is permitted, then everything is permitted

rich discussion on from variants of permission to the meaning of ‘or’…

Ross's paradox

in **D**:

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$p \longrightarrow p \vee q$ (PL)

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p: you mail the letter

q: you burn the letter

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It is obligatory that you mail the letter.

It is obligatory that you mail the letter or you burn the letter.

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It is obligatory that you mail the letter.

It is obligatory that you mail the letter or you burn the letter.

Does the obligation of mailing the letter generate another which can be fulfilled by burning it?

Good Samaritan's paradox

in **D**:

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$p \land q \longrightarrow q$ (PL)

Good Samaritan's paradox

in **D**:

$if\ \vdash p \longrightarrow q\ then \vdash Op \longrightarrow Oq$ (derived derivation rule)

$p \wedge q \longrightarrow q$ (PL)

$$O(p \wedge q) \longrightarrow Op$$

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It is obligatory that Jones help Smith who is being robbed.

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$$O(h \wedge r)$$

we can derive that it is obligatory that Smith is being robbed: Or

Åqvist's paradox of epistemic obligation

The bank is being robbed.
It is obligatory that Jones (the guard) knows that the bank is being robbed.

$$r$$
$$OKr$$

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In epistemic logics, the axiom T usually holds: $Kp \longrightarrow p$
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(works not only with forbidden acts: the guard should also know that the (later) robber is in the bank, and being in a bank is permitted for anyone…)

Problems of deontic conditionals

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Bob’s promising to meet you commits him to meeting you

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$$O(p \to m)$$

Problems of deontic conditionals

Bob's promising to meet you commits him to meeting you

$$O(p \rightarrow m)$$

But in **D**: $if \ \vdash p \longrightarrow q \ then \vdash Op \longrightarrow Oq$ (derived derivation rule)

Problems of deontic conditionals

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$$O(p \to m)$$

But in **D**: $if \ \vdash p \longrightarrow q \ then \vdash Op \longrightarrow Oq$ (derived derivation rule)

So, we have:

$$O\neg p \longrightarrow O(p \longrightarrow q)$$
$$Oq \longrightarrow O(p \longrightarrow q)$$

anything forbidden commits us to everything
for anything obligatory, everything commits us to it

(first noticed by Prior in von Wright's system in 1954)

Problems of deontic conditionals

Bob’s promising to meet you commits him to meeting you

$$p \to Om$$

Problems of deontic conditionals

Bob’s promising to meet you commits him to meeting you

$$p \rightarrow Om$$

Is it better?

Problems of deontic conditionals

Bob's promising to meet you commits him to meeting you

$$p \rightarrow Om$$

Is it better?

No.

Problems of deontic conditionals

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$$p \rightarrow Om$$

Is it better?
No.

we have:

$$\neg p \longrightarrow (p \longrightarrow Oq)$$

anything false would commit us to anything whatsoever
(e.g. since I did not promise you to meet, it would follow that my promising to meet you commits me to buy food)

Problems of deontic conditionals

Bob's promising to meet you commits him to meeting you

$$p \to Om$$

Is it better?
No.

we have:

$$\neg p \longrightarrow (p \longrightarrow Oq)$$
$$Oq \longrightarrow (p \longrightarrow Oq)$$

anything false would commit us to anything whatsoever
(e.g. since I did not promise you to meet, it would follow that my promising to meet you commits me to buy food)
for anything obligatory, everything commits us to it
(e.g. if I'm obligated to call you, then the fact that Magnum has a red Ferrari commits me to call you)

…most important of all paradoxes:

Chisholm’s paradox