ESSLLI 2026

Experimenting with the LogiKEy Framework & Methodology

Day 3: The problem of Contrary-to-Duty: Preference and Dyadic Deontic Logics

Christoph Benzmüller and Luca Pasetto

Based on slides by Réka Markovich

Von Wright’s deontic logic/MDL

very simple, very intuitive

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unfortunately, full of problems

Problems of deontic conditionals

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Bob's promising to meet you commits him to meeting you

$$O(p \to m)$$

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Bob's promising to meet you commits him to meeting you

$$O(p \rightarrow m)$$

But in **D**: $if\ \vdash p \longrightarrow q\ then \vdash Op \longrightarrow Oq$ (derived derivation rule)

So, we have:

$$O\neg p \longrightarrow O(p \longrightarrow q)$$
$$Oq \longrightarrow O(p \longrightarrow q)$$

anything forbidden commits us to everything
for anything obligatory, everything commits us to it

(first noticed by Prior in von Wright's system in 1954)

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$$p \to Om$$

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$$p \rightarrow Om$$

Is it better?

No.

we have:

$$\neg p \longrightarrow (p \longrightarrow Oq)$$
$$Oq \longrightarrow (p \longrightarrow Oq)$$

anything false would commit us to anything whatsoever

(e.g. since I did not promise you to meet, it would follow that my promising to meet you commits me to buy food)

for anything obligatory, everything commits us to it

(e.g. if I'm obligated to call you, then the fact that Magnum has a red Ferrari commits me to call you)

Problems of deontic conditionals

So standard (monadic) deontic logic cannot handle conditional obligations?

…most important of all paradoxes:

Chisholm’s paradox

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(A) It ought to be that Jones goes to the assistance of his neighbours;
(B) It ought to be that, if Jones goes to the assistance of his neighbours, then he tells them he is coming;
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(A)Primary obligation

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But if we try to formalize them in **D**…

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(D_0) $\neg g$

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$(B_0)\ g \to \bigcirc t$	$(B_1)\ \bigcirc(g \to t)$	$(B_2)\ \bigcirc(g \to t)$	$(B_3)\ g \to \bigcirc t$
$(C_0)\ \bigcirc(\neg g \to \neg t)$	$(C_1)\ \neg g \to \bigcirc \neg t$	$(C_2)\ \bigcirc(\neg g \to \neg t)$	$(C_3)\ \neg g \to \bigcirc \neg t$
$(D_0)\ \neg g$	$(D_1)\ \neg g$	$(D_2)\ \neg g$	$(D_3)\ \neg g$
We cannot derive any further obligation	Inconsistent	Not independent	Not independent

Chisholm's paradox

(A$_1$) $\bigcirc g$
(B$_1$) $\bigcirc(g \to t)$
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(D$_1$) $\neg g$

Proposition 1. *The set* $\Gamma = \{A_1, B_1, C_1, D_1\}$ *is not satisfiable, and thus it is inconsistent.*

Proof. Assume there exist a relational model $M = (W, R, V)$ and a state $s_1 \in W$ such that

- $M, s_1 \vDash \bigcirc g$
- $M, s_1 \vDash \bigcirc(g \to t)$
- $M, s_1 \vDash \neg g \to \bigcirc\neg t$
- $M, s_1 \vDash \neg g$.

From $M, s_1 \vDash \neg g$ and $M, s_1 \vDash \neg g \to \bigcirc\neg t$ we deduce $M, s_1 \vDash \bigcirc\neg t$. By seriality we know there is $s_2 \in W$ such that $s_1 R s_2$. By the definition of $\bigcirc$ we know $M, s_2 \vDash \neg t$, $M, s_2 \vDash g$, $M, s_2 \vDash g \to t$, which is a contradiction.

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$(\mathrm{B}_0)\ g \rightarrow \bigcirc t$	$(\mathrm{B}_1)\ \bigcirc(g \rightarrow t)$	$(\mathrm{B}_2)\ \bigcirc(g \rightarrow t)$	$(\mathrm{B}_3)\ g \rightarrow \bigcirc t$
$(\mathrm{C}_0)\ \bigcirc(\neg g \rightarrow \neg t)$	$(\mathrm{C}_1)\ \neg g \rightarrow \bigcirc \neg t$	$(\mathrm{C}_2)\ \bigcirc(\neg g \rightarrow \neg t)$	$(\mathrm{C}_3)\ \neg g \rightarrow \bigcirc \neg t$
$(\mathrm{D}_0)\ \neg g$	$(\mathrm{D}_1)\ \neg g$	$(\mathrm{D}_2)\ \neg g$	$(\mathrm{D}_3)\ \neg g$
We cannot derive any further obligation	Inconsistent		

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(B_0) $g \rightarrow \bigcirc t$	(B_1) $\bigcirc(g \rightarrow t)$	(B_2) $\bigcirc(g \rightarrow t)$	(B_3) $g \rightarrow \bigcirc t$
(C_0) $\bigcirc(\neg g \rightarrow \neg t)$	(C_1) $\neg g \rightarrow \bigcirc \neg t$	(C_2) $\bigcirc(\neg g \rightarrow \neg t)$	(C_3) $\neg g \rightarrow \bigcirc \neg t$
(D_0) $\neg g$	(D_1) $\neg g$	(D_2) $\neg g$	(D_3) $\neg g$
We cannot derive any further obligation	Inconsistent	Not independent	Not independent

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P2: p
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FACTUAL DETACHMENT

2)
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DEONTIC DETACHMENT

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crucial part of normative reasoning is about what to be done after a violation

the obligation of apologizing if hurting someone
the obligation of paying for damages we caused
the rules of how to act once the crew has failed to follow the primary rules at a nuclear plant...

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CTD: how things ought to be under deontically unacceptable conditions

We should handle propositions that are relatively-acceptable:
acceptable relative to the assumption that the states will be those unacceptable ones where the specific violation conditions hold

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$$P(p/c)$$

axioms:

$$P(p/c) \vee P(\neg p/c)$$

$$P(p \wedge q/c) \leftrightarrow P(p/c) \wedge P(q/c \wedge p)$$

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Dyadic Deontic Logic (DDL) can be described as a generalisation of MDL

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$$\phi ::= p \mid \neg\phi \mid \phi \wedge \phi \mid \Box\phi \mid \bigcirc(\phi/\phi)$$

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$P(\psi/\phi)$ *("ψ is permitted, given ϕ") is short for $\neg \bigcirc (\neg\psi/\phi)$, $\bigcirc\phi$ ("ϕ is unconditionally obligatory") and $P\phi$ ("ϕ is unconditionally permitted") are short for $\bigcirc(\phi/\top)$ and $P(\phi/\top)$, respectively. $\Diamond\phi$ is short for $\neg\Box\neg\phi$. Other Boolean connectives are defined as in the previous chapter.*

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ranking states based on the number of obligations they violate:
the more obligations are violated in a given state, the farther from ideality this state is

Dyadic Deontic Logic

Preference models

Definition *A preference model $M = (W, \succeq, V)$ is a tuple where:*

- *W and V are as before.*
- *$\succeq$ is a binary relation over W ordering the states according to their relative goodness. $s \succeq t$ is read as "state s is at least as good as state t".*

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one state better than another
iff
it violates only a subset of the explicit obligations of the
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Dyadic Deontic Logic

Preference models

Definition (Equal goodness). *Two states s and t are equally good (notation: $s \simeq t$) if $s \succeq t$ and $t \succeq s$.*

Definition (Incomparability). *Two states s and t are incomparable (notation: $s \| t$) if $s \not\succeq t$ and $t \not\succeq s$.*

Definition (Strict betterness). *The strict relation $\succ$ induced by $\succeq$ is defined by: $s \succ t$ iff $s \succeq t$ and $t \not\succeq s$. For $s \succ t$, read "s is strictly better than t".*

Dyadic Deontic Logic

Preference models

Definition (Satisfaction). *Given a preference model $M = (W, \succeq, V)$ and a state $s \in W$, we define the satisfaction relation $M, s \vDash \phi$ as before except for the following two new clauses*

- $M, s \vDash \Box\phi$ *iff, for all* $t \in W$, $M, t \vDash \phi$
- $M, s \vDash \bigcirc(\psi/\phi)$ *iff* $best(\|\phi\|) \subseteq \|\psi\|$

where $\|\phi\|$ is the truth-set of ϕ, viz. the set of all the ϕ-states, and $best(\|\phi\|)$ the subset of those that are best according to $\succeq$.

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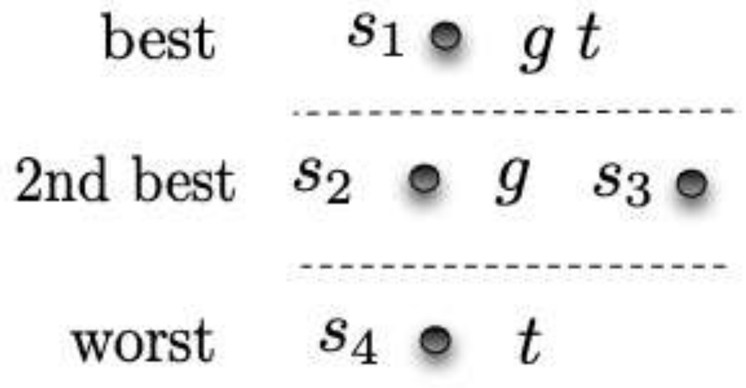

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Definition (Optimality). *For state s to qualify as best among the ϕ-states, s must be optimal under $\succeq$: s makes ϕ true, and s is at least as good as any other states t making ϕ true. Formally:*

$$\begin{aligned} \mathrm{best}_{\succeq}(\|\phi\|) &= \mathrm{opt}_{\succeq}(\|\phi\|) \\ &= \{s \in \|\phi\| \mid \forall t \ \ (t \vDash \phi \rightarrow s \succeq t) \end{aligned}$$

Dyadic Deontic Logic

Proof Theory

Definition **E** *is the proof system consisting of the following axiom schemas and rule schemas.*

ϕ, *where* ϕ *is a tautology from PL*	**(PL)**	
$\Box(\phi \to \psi) \to (\Box\phi \to \Box\psi)$	**($\Box$-K)**	
$\Box\phi \to \phi$	**($\Box$-T)**	$\Box$ is an S5 modality
$\neg\Box\phi \to \Box\neg\Box\phi$	**($\Box$-5)**	
$\bigcirc(\psi \to \chi/\phi) \to (\bigcirc(\psi/\phi) \to \bigcirc(\chi/\phi))$	**(COK)**	COK is is the dyadic analogue of the K axiom
$\bigcirc(\phi/\phi)$	**(Id)**	Id is the dyadic analogue of the principle of identity
$\bigcirc(\chi/(\phi \wedge \psi)) \to \bigcirc((\psi \to \chi)/\phi)$	**(Sh)**	Sh is deontic analogue of the deduction theorem
$\bigcirc(\psi/\phi) \to \Box\bigcirc(\psi/\phi)$	**(Abs)**	Absoluteness: ranking is not made relative to states
$\Box\psi \to \bigcirc(\psi/\phi)$	**(Nec)**	Nec is the analogue of the principle of necessitation
$\Box(\phi \leftrightarrow \psi) \to (\bigcirc(\chi/\phi) \leftrightarrow \bigcirc(\chi/\psi))$	**(Ext)**	Ext: principle of replacement of logical equivalents
If $\vdash \phi$ *and* $\vdash \phi \to \psi$ *then* $\vdash \psi$	**(MP)**	
If $\vdash \phi$ *then* $\vdash \Box\phi$	**($\Box$-Nec)**	

Dyadic Deontic Logic

Proof Theory

Definition **F** *is the proof system obtained by supplementing* **E** *with*

$$\Diamond\phi \to (\bigcirc(\psi/\phi) \to P(\psi/\phi)) \qquad (\bigcirc\text{-}\mathbf{D}^\star)$$

$\bigcirc$-D* is the dyadic analogue of the $\bigcirc$-D axiom. Intuitively, it rules out the possibility of conflicts between conditional obligations, for a "consistent" ϕ.

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Preference models

Invalidities: a price to pay?

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Preference models

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Dyadic Deontic Logic

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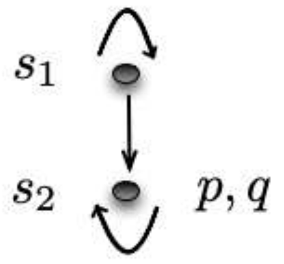
Proof. Let $M = (W, \preceq, V)$ be such that $W = \{s_1, s_2\}$, $\preceq$ is the reflexive closure of $\{(s_1, s_2)\}$, $V(p) = V(q) = \{s_2\}$.



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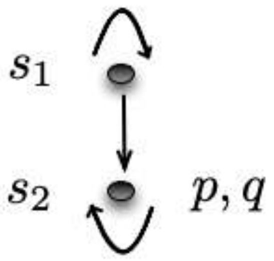

$s_2 \models p$

$\mathrm{opt}_{\succeq}(\|p\|) = \{s_2\} \subseteq \|q\| = \{s_2\} \Rightarrow s_2 \models \bigcirc(q/p)$

$\mathrm{opt}_{\succeq}(\|\top\| = \{s_1\}) \not\subseteq \|q\| \Rightarrow s_2 \not\models \bigcirc q$